Accepted Manuscript

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PII: S0263-8223(16)32320-0
DOI: http://dx.doi.org/10.1016/j.compstruct.2016.11.083
Reference: COST 8044

To appear in: *Composite Structures*

Received Date: 27 October 2016
Revised Date: 23 November 2016
Accepted Date: 25 November 2016

Please cite this article as: Tomczyk, B., Szczerba, P., Tolerance and asymptotic modelling of dynamic problems for thin microstructured transversally graded shells, *Composite Structures* (2016), doi: http://dx.doi.org/10.1016/j.compstruct.2016.11.083

Tolerance and asymptotic modelling of dynamic problems for thin microstructured transversally graded shells

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Abstract

The objects of consideration are thin linearly elastic Kirchhoff-Love-type open circular cylindrical shells having a functionally graded macrostructure and a tolerance-periodic microstructure in circumferential direction. The first aim of this contribution is to formulate and discuss a new non-asymptotic averaged model for the analysis of selected dynamic problems for these shells. As a tool of modelling we shall apply *the tolerance averaging technique*. Contrary to the starting exact shell equations with highly oscillating, non-continuous and tolerance-periodic coefficients, governing equations of the proposed *tolerance model* have continuous and slowly varying coefficients depending also on a cell size. Hence, an important advantage of this model is that it makes it possible to study the effect of a microstructure size on the global shell dynamics (*the length-scale effect*). The second aim is to derive and discuss a certain asymptotic model being independent of a microstructure size. It will be shown that in the framework of the tolerance model *not only the fundamental lower*, *but also the new additional higher-order free vibration frequencies can be derived and analysed*.

Keywords: Thin transversally graded shells; Tolerance-periodic microstructure, Tolerance modelling; Asymptotic modelling, Dynamic problems

1. Introduction

Thin linearly elastic Kirchhoff-Love-type open circular cylindrical shells with a tolerance-periodic microstructure in circumferential direction are analysed. It means that *on the microscopic level*, the shells under consideration consist of many small elements. These elements, called *cells*, are treated as thin shells. It is assumed that *the adjacent cells are nearly identical, but the distant elements can be very different*. An example of such shell is shown in Fig. 1. At the same time, the shells have constant structure in axial direction. On the microscopic level, the geometrical, elastic and inertial properties of these shells are determined by highly oscillating, non-continuous and *tolerance-periodic functions* in circumferential direction. By *tolerance periodic functions* we shall mean functions which in every cell can be approximated by periodic functions.

On the other hand, *on the macroscopic level*, the averaged (effective) properties of the shells are described by functions being *smooth* and *slowly varying* along circumferential direction. It means that the tolerance-periodic shells under consideration can be treated as made of *functionally graded materials* (FGM), cf. Suresh and Mortensen [1], and called *functionally graded shells*. Moreover, since effective properties of the shells are graded in direction normal to interfaces between constituents, this gradation is referred to as *the transversal gradation*.

The dynamic problems of such shells are described by partial differential equations with highly oscillating, tolerance-periodic and non-continuous coefficients, so these equations are too complicated to apply to investigations of engineering problems. To obtain averaged equations with continuous and slowly