Composite Structures 166 (2017) 229–238

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Parametric instabilities of variable angle tow composite laminate under axial compression

Surya Samukham, Gangadharan Raju *, C.P. Vyasarayani

Department of Mechanical and Aerospace Engineering, Indian Institute of Technology Hyderabad, 502205, India

A R T I C L E I N F O

Article history:
Received 2 November 2016
Revised 20 December 2016
Accepted 13 January 2017
Available online 23 January 2017

Keywords:
Dynamic instability
Variable angle tow composites
Bolotin's approach
First order shear deformation theory

A B S T R A C T

Dynamic instability of a variable angle tow (VAT) composite plate subjected to periodic in-plane compressive load is investigated using finite element analysis. First order shear deformation theory was used to model the VAT laminate and the effect of fiber angle orientation on the instability behavior of VAT laminates is studied. Unlike straight fiber composites, the pre-buckling problem of VAT laminate is solved initially to obtain the plate in-plane stress distribution due to the applied uniform compression along the edges. Subsequently, the evaluated stress distributions were used in the equations governing the dynamic instability of VAT laminates. The dynamic instability regions of VAT laminates are determined using Bolotin's first order approximation. The dynamic instability results of VAT plates are evaluated for linear fiber angle distribution and their performances are then compared with straight fiber laminates. Effect of the fiber angle orientation, load parameters, boundary conditions, orthotropy ratio and aspect ratio on the dynamic instability regions of VAT laminate are investigated in detail.

1. Introduction

Advancement in automation of composite manufacturing has led to new techniques that enable fabrication of composite structures with greater repeatability, high production rate and less defects. In addition, automation allow options for fiber steering and provides designer with improved tailor-ability options when compared to straight fiber composites. The composites manufactured by steering the fiber tow paths curvilinearly in the plane of the lamina, and are known as variable angle tow (VAT) composites. By allowing the fiber orientation to change in the plane of the structure, the stiffness of the structure can be varied, and this results in a redistribution of loads in critical regions. VAT composites are manufactured using automated-fiber-placement (AFP), automated-tape-laying (ATL) and continuous-tow-shearing (CTS) techniques. VAT composites are currently being used in aerospace and wind turbine applications [1,2].

Hyer and Lee [3] were among the first to investigate the influence of curvilinear fiber around a cutout of a flat composite plate. They proposed that better designs can be developed by aligning the fibers along the principal directions of the stress field. Subsequently, they worked on designing a VAT plate with maximum buckling load [3]. They determined the optimal fiber orientation angles in different regions of the laminate using a combination of the finite element method (FEM), sensitivity analysis, and optimization techniques. To simplify the fabrication process of VAT laminates, Gurdal and Olmedo [4] introduced a linear fiber angle variation along the length of the composite laminate. Later, Gurdal et al. [5] studied the buckling of VAT panels by varying the stiffness in perpendicular directions with respect to loading. The buckling coefficient was computed using the Rayleigh–Ritz method. Alhajahmad et al. [6] studied the nonlinear pressure pillowing problem of fuselage skin panels. They concluded that VAT panels could carry more load compared to straight fiber composite panels. By using FEM, Abdalla et al. [7] designed VAT plates to maximize the first natural frequency. Setoodeh et al. [8] used a reciprocal approximation technique to optimize VAT panels for maximum buckling load. They employed a conforming bilinear FEM for buckling analysis of VAT panels. Raju et al. [9] analyzed the buckling and post-buckling behavior of VAT plates using differential quadrature method (DQM). They found that DQM gave reasonably accurate solutions with fewer grid points compared to FEM [10]. Honda and Narita [11], Akhavan and Ribeiro [12] studied vibration of VAT panels using a higher order plate theory combined with FEM. They used optimization techniques to design VAT panels for better structural performance. Apart from FEM and DQM, numerous semi-analytical approaches like, Rayleigh–Ritz, Galerkin, and perturbation

* Corresponding author.
E-mail address: gangadharanr@iith.ac.in (G. Raju).

http://dx.doi.org/10.1016/j.compstruct.2017.01.044
0263-8223/

methods were also used to solve the post-buckling problem of composite plates [13–15].

In addition to in-plane buckling and post-buckling analysis, dynamic stability analysis is equally important while designing VAT laminates. Structural members like beams, plates and shells are often subjected to periodic in-plane loads and become dynamically unstable for certain combination of load amplitude and the excitation frequency of the structure. A comprehensive study on the dynamic/parametric instability behavior of various elastic systems has been carried out by Bolotin [16]. After that some researchers have used Bolotin's approach to investigate the dynamic instability of structures [17–20]. Birman [21] studied the dynamic stability of an unsymmetrical cross-ply simply supported rectangular laminate under a harmonically varying bi-axial compression load and principal dynamic instability regions (DIR) were determined analytically. Srinivasan and Chellapandi [22] investigated the dynamic stability of thin laminated plates under harmonically varying uni-axial stress systems by using the semi-analytical finite strip method (FSM).

All the above mentioned studies [17–22] used classical laminated plate theory (CLPT) for their analysis. However, while studying composite laminates, transverse shear effects are significant, and the use of shear deformation plate theory (SDPT) provides better results when compared to CLPT. Therefore, Bert and Birman [23], Chen and Yang [24], Moorthy et al. [25], Chattopadhyay and Radu [26], Wang and Dawe [27] have used the SDPT in the dynamic instability analysis composite laminates. Extensive results were presented on the effects of different parameters like damping, thickness-to-length ratio, anisotropy, boundary conditions, the number of layers and lamination angles on dynamic stability of the composite laminates. Numerical methods such as the method of multiples scales and Lyapunov direct methods have also been employed for the dynamic stability analysis of composite laminates [28,29]. Sahu and Datta [30] used FEM to study parametric instability in laminated composite plates subjected to a non-uniform in-plane periodic loads.

From the literature, it is evident that a large amount of work has been conducted on the parametric instability of straight fiber composites (for example see the review paper by Sahu and Datta [31]). However, it appears that no work has been done on the parametric instability of VAT plates. In this work, dynamic instability analysis of symmetric VAT laminate subjected to periodic uni-axial compression is studied. Dynamic instability boundaries can be obtained by using the method of multiple scales, perturbation analysis, Lyapunov exponents [32], Bolotin's approach or Floquet's theory. However, due to the ease of implementation and computational efficiency, we resort to Bolotin's in this work for generating instability boundaries. The analysis is carried out by modeling the VAT laminate using Reissner–Mindlin plate theory combined with FEM [33]. The laminate was discretized using a four-noded quadrilateral iso-parametric element with 5 degrees of freedom per node. The effect of tow steering on the dynamic instability behavior under periodic uni-axial compression loading is investigated, and their parametric resonance behavior is compared with straight fiber composites.

The paper is organized as follows: In Section 2 we discuss the modeling of VAT laminates. In Section 3, using FEM, the stiffness and mass matrices of VAT laminates are derived. In Section 4 the Bolotin's first order approximation for obtaining stability boundary under parametric loading is discussed. In Section 5, we validate our model with results from a commercial finite element software and from results from the literature. Further, the influence of static load parameter, orthotropy, in-plane and plate boundary conditions, and aspect ratio on the dynamic instability regions of the VAT laminates are presented. Finally, we conclude our findings in Section 6.

2. Modeling of variable angle tow (VAT) laminates

The terminology, variable angle tow refers to the composite panels with continuously varying in-plane fiber orientation. This results in a change of laminate stiffness as a function of $x-y$ coordinates. Unlike conventional straight fiber composites, VAT laminates cannot be described by a single fiber orientation angle. Therefore, the fiber orientation of a VAT lamina are usually represented using polynomials, splines, and NURBS. In this work, Lagrange polynomials are used to describe linear fiber angle variation [4] in one direction of the laminate and can be expressed as follows:

$$\psi(x) = \phi + \frac{2(T_1 - T_0)}{a}|x| + T_0, \tag{1}$$

where, T_0 is the fiber orientation angle at the center of the panel $(x=0)$ and T_1 is the fiber orientation angle at the panel end $(x=\pm a/2)$ and ϕ is the angle of rotation of the fiber path (see Fig. 1).

In this work, Reissner–Mindlin plate theory [33] is used to model the VAT laminates. A composite plate with its mid-plane represented by (x, y) coordinate and the thickness along the z axis is assumed. Based on Reissner–Mindlin plate theory, the displacement fields (u, v, w) are expressed as:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\theta_x(x,y), \\ v(x,y,z) &= v_0(x,y) - z\theta_y(x,y), \text{ and} \\ w(x,y,z) &= w_0(x,y). \end{aligned} \tag{2}$$

Here, w_0, θ_x and θ_y are the mid-plane kinematic variables for bending and u_0 and v_0 are the in-plane displacements for membrane behavior. The strain–displacement relationship for the laminate can be expressed as:

$$\boldsymbol{\epsilon} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\epsilon}_m \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} -z\boldsymbol{\epsilon}_b \\ \boldsymbol{\epsilon}_s \end{Bmatrix}. \tag{3}$$

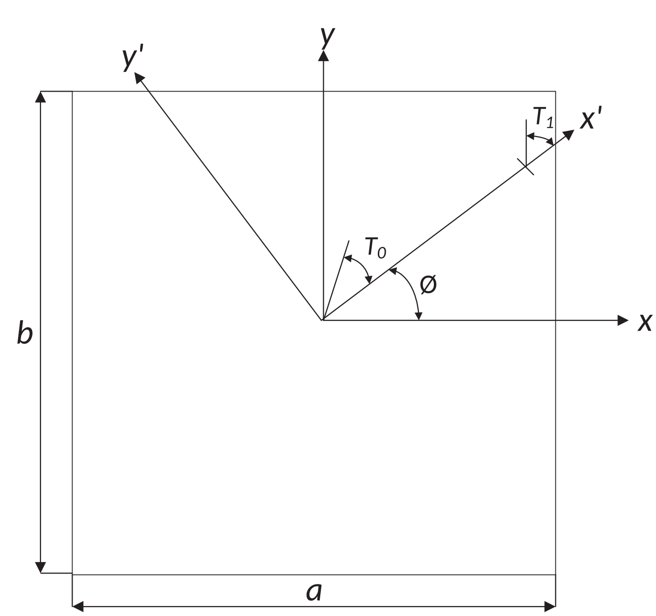

Fig. 1. Components used in definition of reference fiber path.