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A four-node facet shell element for laminated shells based on the third order zigzag theory

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ABSTRACT

In this work, we develop a facet shell element for deep laminated shells, by extending a successful four-node quadrilateral element for laminated plates based on the efficient third order zigzag theory. The obstacle course test comprising of three standard problems is undertaken to examine its performance for various modes of shell behavior. The absence of shear and membrane locking problems is established through the analysis of ultra thin shells. The accuracy of the element is assessed for the static and free vibration responses of composite and sandwich shells in comparison with the three dimensional elasticity solutions. In terms of accuracy, computational efficiency and robustness, the present element is shown to give better performance than various classical and recent finite elements considered in this study. In the case of sandwich shells, for which the equivalent single layer theories showed a high level of error, the present element is shown to yield more accurate results than even the higher-order sandwich shell theories that have been developed specifically for the three-layer sandwich shells.

1. Introduction

The growing popularity and success of laminated structures in various industries particularly in aerospace, naval, automobile and space applications has indeed compelled scientists and engineers to analyse these structures using models which are more accurate and computationally efficient than the existing ones. This is evident from the large number of publications (∼515) that have appeared in the past decade on the computational models for static and dynamic analysis of composite laminated shells, as listed in the review article of Qatu et al. [1,2]. The realization of the scope and versatility of the finite element method (FEM) in 60 s resulted in its growing popularity, and a large number of parallel studies were conducted for the development of finite elements for analysis of isotropic and laminated shell structures. The three dimensional (3D) solid elements are often computationally inefficient for modeling layered structures as the problem size increases in proportion with the number of layers. Hence, finite element models based on two-dimensional shell theories have been developed.

Several review articles have been published periodically on the development of shell finite elements based on 2D models [3–6]. The triangular facet shell elements [7,8], the doubly curved triangular shell elements based on the Novozhilov's shallow shell theory [9–11], the rectangular doubly curved shallow shell element [12] and the degenerated curved shell element [13] are among the earliest shell elements developed for the analysis of isotropic shells. Apparently the first FE model for laminated anisotropic shells of general shapes is due to Thompson and Bert [14], which was developed based on the classical laminate theory (CLT). This element was, however, restricted to shallow shells. The CLT completely neglects the shear deformation effect, which is known to be very significant in fiber reinforced polymer (FRP) composites due to their low shear modulus to longitudinal modulus ratio. Also, this theory requires C^1-continuity of the deflection variable, which is difficult to be satisfied in a quadrilateral element. For these reasons, elements based on shear deformable theories, requiring C^0-continuity of interpolation functions were developed. Panda and Natarajan [15] presented an eight-node doubly curved quadratic shell element for laminated composite shells, by extending the Ahmad's degenerate shell element for isotropic shells [13]. Shell elements developed based on the FSDT include the linear and quadratic elements presented by Reddy [16] and Chakravorty et al. [17]. However, the early FSDT based C^0-continuous elements suffered from problems such as shear/membrane locking in thin shells and spurious zero energy modes for which improvements such as the so called mixed-enhanced formulation [18] and the mixed interpolation of tensorial components (MITC) [19] have been presented. A major drawback of the FSDT for laminated structures is

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its strong dependence on shear correction factors, whose accurate estimation for anisotropic laminates is not trivial [20]. This has motivated the development of higher order theories (HOTs) that do not require shear correction factors. C^0-continuous four-node, nine-node Lagrangian and eight-node serendipity shell elements based on HOTs have been presented for laminated composite cylindrical shells [21,22] and doubly curved shells [23,24].

The CLT, FSDT and the HOTs belong to the category of equivalent single layer (ESL) theories in which the displacements are approximated to follow a single global variation across all layers of the laminate. This assumption makes the number of primary displacement variables independent of the number of layers in the laminate, and consequently the theories are computationally efficient. But, they can give very erroneous results for moderately thick and even thinner laminates having high anisotropy and inhomogeneity, since they do not account for the slope discontinuity in the through-thickness variations of the inplane displacements, at the interfaces between adjacent layers of different material properties, as observed from the 3D elasticity solutions [25]. To incorporate the slope discontinuity of the inplane displacement field, layerwise theories (LWT's) have been developed for laminated shells, wherein the displacements are assumed to follow a linear or higher order variation across each layer separately [26]. Several triangular [27,28] and quadrilateral [22,29–31] shell elements have been developed based on the LWTs for linear and nonlinear analysis of laminated shells. The LWTs are, however, computationally inefficient like the full 3D FE analysis, since the number of primary variables increases proportionally with the number of layers.

In order to retain the computational efficiency of the ESL theories, while allowing for the layerwise distortion of the normal to the shell surface, efficient layerwise theories (ELTs) have been developed. In these theories, the layerwise function of the thickness coordinate for the inplane displacements is first superimposed with a global function, but the number of unknowns is finally made layer-independent by imposing the conditions of transverse shear stresses at the layer interfaces and the prescribed shear traction conditions at the top and bottom surfaces of the laminate. Such theories were originally pioneered by Ambartsumyan [32] for static and by Rath and Das [33] for dynamic analysis, but were not followed up for a long time. Their reformulations with some variations were presented by Di Sciuva [34], Cho and Parmerter [35] and Shu and Sun [36] for anisotropic laminated plates. The third order zigzag theory (ZIGT) [35,36] has been shown to yield very accurate results in comparison with the exact 3D elasticity solutions for static, dynamic and buckling response of thin to moderately thick highly inhomogeneous composite and sandwich plates [37,38]. It was extended to shallow laminated shells by Shu [39] and deep cylindrical shells under thermomechanical loading by Dumir et al. [40].

The first FE implementation of the Ambartsumyan-Rath-Das type ELT for shell structures is due to Beakou and Touratier [41], who developed a C^1-continuous rectangular element for shallow laminated shells. Dau et al. [42] has presented a C^1-continuous six-node triangular element for the dynamic analysis of general multilayered shell structures, based on an ELT with a sinusoidal global variation and a linear piecewise zigzag variation for the inplane displacements. Cinefra and Carrera [6] have presented a nine-node element for static analysis of multilayered cylindrical shells using a unified formulation containing various ESL and layerwise theories and an ELT using Murakami's zigzag function [43]. Eijo et al. [44] and Versino et al. [45] have presented four-node elements for laminated composite plates and doubly curved shells, respectively, based on a refined zigzag theory. In this theory, a zigzag function similar to the Murukami's function is used, while relaxing the requirement of interfacial continuity of transverse shear stress. The formulation, thus, faces the problem of shear locking which is addressed by using the MITC interpolation of assumed natural strains.

The third order ZIGTs of the type presented by Cho and Parmerter [35] and Shu and Sun [36] requires C^1-continuity, which poses difficulty in developing simple quadrilateral elements. Kumar et al. [46] have presented a nine-node C^0-continuous isoparametric quadrilateral element for shallow laminated shells based on the ZIGT of Cho and Parmerter [35], by treating the derivatives of the mid-surface deflection, $\frac{\partial w_0}{\partial x}$ and $\frac{\partial w_0}{\partial y}$, as independent variables. This, however, amounts to modifying the original theory and relaxing the conditions of zero shear tractions at the top and bottom surfaces of the laminate. Kapuria and Kulkarni [47] developed a four-node quadrilateral element for multilayered anisotropic plates, based on the ZIGT of Shu and Sun [36], wherein the requirement of C^1-continuity has been circumvented by using the so called improved discrete Kirchhoff (IDKQ) technique, originally proposed by Jeyachandrabose et al. [48], for plate bending elements. This element was shown to be shear locking free, and yield very accurate results in comparison with the 3D elasticity based analytical and FE solutions for static and dynamic response of composite and sandwich plates of different lay-ups, shapes and boundary conditions. Comparisons with the other existing elements established its superiority, in terms of simplicity, accuracy, computational efficiency and robustness. Inspired by the success of this discrete Kirchhoff zigzag theory (DKZIGT) plate element, a four-node quadrilateral element for doubly curved shallow laminated shells was developed by Yasin and Kapuria [49], which again was shown to yield very accurate results for a variety of shell geometries, but the formulation and results were presented only for shallow shells of rectangular planform in which only the projection on the planform surface was modeled, and not the curved shell surface. No study has been presented so far on the analysis of general laminated shell structures (without the shallowness restriction) based on this accurate ZIGT.

In this paper, a four-node quadrilateral facet element based on the efficient ZIGT is presented for the static and dynamic analysis of general laminated shell structures. The original ZIGT based quadrilateral element of Kapuria and Kulkarni [47] for laminated plates has four nodes with seven degrees of freedom per node. This element has been generalized to deep shells, by using an oblique local reference plane for each element, and employing coordinate transformation. The local reference plane takes care of the non-coplanar nodes that may occur in the meshing of shells having twisting curvature, making the element suitable to model shells of all shapes. The local coordinate axes are chosen such that they are nearly parallel to element edges. For the purpose of transformation of mid-surface rotation and shear rotation variables of element, two additional drilling degrees of freedom (DOFs) are introduced. The element mass and stiffness matrices and the load vector are derived using the Hamilton's principle. The element is also first of its kind for general shells based on a theory requiring C^1 continuity even for isotropic shells. The accuracy of the element is assessed by comparing with the analytical 3D elasticity solutions and other 2D theory based solutions for stress and free vibration response of laminated deep shells.

2. Local coordinate system

For a four-node quadrilateral element on a shell surface, the four corner nodes with position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ and $\mathbf{r}_4$, respectively, may not be coplanar. To maintain the geometric compatibility in such cases, the element is projected to the plane that passes through its midpoint $\mathbf{r}_c = (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)/4$and is parallel to the diagonals $\mathbf{d}_1 = \mathbf{r}_3 - \mathbf{r}_1$ and $\mathbf{d}_2 = \mathbf{r}_2 - \mathbf{r}_4$ [50]. The local axes (x, y) for the element are chosen such that they are nearly parallel to the