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Theory of thin-walled functionally graded sandwich beams with single and double-cell sections

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ABSTRACT

In the present work, a computational model has been presented to study the flexural and torsional analyses of thin-walled functionally graded sandwich beams with single and double-cell sections. The analysis model is based on the Euler-Bernoulli beam theory and includes the effects of elastic couplings and constrained warping. The mechanical properties of beam such as Young's and shear moduli are assumed to continuously vary in the thickness direction based on the power law distribution of volume fraction of ceramic or metal. To solve the flexural and torsional problems of functionally graded material (FGM) sandwich box beams, the finite beam element considering Hermite cubic interpolation polynomials is employed with the scope to discretize the governing equations. The theory is validated against the analytical solutions and the finite element analysis results for beams with single and double-cell box sections. Three types of material distribution are considered to investigate the effects of gradient index, thickness ratio of ceramic, material ratio, and width-to-height ratio on the various rigidities of cross-section of FGM sandwich box beams. Numerical results show that the above mentioned effects play important role on the structural responses of FGM sandwich box beams.

1. Introduction

Recently, the use of new classes of composite materials known as functionally graded materials (FGMs) has found increasing applications in a variety of engineering fields such as civil, aerospace, defense and marine industries. FGMs have the merit to avoid the undesirable stress discontinuity existing between two surfaces in laminated composites and have the stronger mechanical performance of metal phase and the better thermal resistance of ceramic phase. Meanwhile, thin-walled beams with closed cross-sections are one of the useful structural members due to their high specific stiffness in bending and torsion. Based on these aspects, thin-walled box beams made of FGMs become emerged as primary load bearing structures in the construction of gas turbines for higher power aircraft engines, helicopter drive applications and variety of aerospace structural systems.

Up to the present, a considerable amount of research has been done in the field of analysis of composite beams using the refined methods. Filippi et al. [1] presented a new class of refined box beam theories based on Chebyshev polynomials for static and dynamic analyses of composite structures. The Carrera Unified Formulation (CUF) was adopted to obtain higher-order beam models. Filippi et al. [2] used 1D CUF to perform static analysis of FGM beam. The hierarchical feature of CUF allowed one to automatically generate an infinite number of displacement theories that might include any kind of functions of the cross-section coordinates. Recently, the layer-wise theories that made use of higher-order zig-zag functions defined over fictitious/mathematical layers of the cross-section area were proposed by Filippi and Carrera [3] within the CUF framework. Carrera et al. [4] examined the static response of a variety of thin-walled laminated beams with closed profile by means of CUF and special attention has been paid to single- and multi-cell beams. Two different classes have been formulated by expanding the unknown kinematic variables on the beam cross-section and they have been referred to as TE (Tayor Expansion) and LE (Lagrange Expansion).

During the past decade, a lot of research effort has been made to model and analyze the thin-walled FGM beams especially with box sections, but most of them are confined to single-cell sections. Mashat et al. [5] employed 1D finite elements based on various displacement theories to perform free vibration analysis of FGM box beam. The implementation of the finite elements in accordance with CUF made it possible to consider a great variety of structures and boundary conditions. Librescu et al. [6] presented the thermoelastic modeling, vibration and instability of spinning

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thin-walled box beams made of functionally graded ceramic-metal based materials. In terms of a simple power law distribution, a continuously graded variation in composition of the ceramic and metal phases across the beam wall thickness is implemented. Librescu et al. [7] also considered the rotating turbomachinery blades and thin-walled beam structures spinning about their longitudinal axis in a high temperature environment. A study devoted to the vibration and stability behavior of FGM spinning circular cylindrical thin-walled beams was presented by Oh et al. [8]. The implications played by the conservative and gyroscopic forces of their instability behavior have been highlighted. Fazelzadeh and Hosseini [9] developed a thin-walled FGM box beam which was used as rotating blades in turbomachinery under aerothermoelastic loading. The governing equations include the effects of presetting angle, secondary warping, temperature gradient through the wall thickness and rotational speed. The differential quadrature vibration analysis of a rotating thin-walled FGM blade operating under the high temperature supersonic gas flow was carried out by Fazelzadeh et al. [10]. In their study, the quasi-steady aerodynamic pressure loadings and steady wall temperature assumptions are made. Piovan and Machado [11] investigated the dynamic stability behavior of thin-walled FGM box beams subjected to axial external force considering the influence of non-conventional effects. The Galerkin's and Bolotin's methods were used to discretize the governing equations and to determine the regions of dynamic stability, respectively. The free vibration of FGM box beam was performed by Ziane et al. [12] based on the formulation of an exact dynamic stiffness matrix. The Newton's eigenvalue iteration method was used to obtain frequencies. They [13] focused on the analysis of lateral-torsional buckling for simply supported thick and thin-walled FGM box beams under uniformly distributed loads. Recently, Lanc et al. [14] investigated the buckling and post-buckling behavior of FGM box beams. The non-linear displacement field of thin-walled cross-section was adopted in order to insure the geometric potential of semitantential type for internal bending and torsional moments.

The existing literature reveals that even though a significant studies have been conducted on the development of improved theories for the analysis of FGM box beams, to the best of the authors' knowledge, the flexural and torsional analyses of thin-walled FGM beams with double-cell sections are done by no researchers till date. Moreover, there still has been no study of investigating the warping restraint effect on the mixed torsional problems of FGM beams with closed cross-sections in the literature. The warping effect is important for beams with open-sections and not negligible for beams with closed-sections with elastic couplings [15].

The objective of this paper is to present a theoretical beam model for the flexural and torsional analyses of thin-walled FGM sandwich beams with single and double-cell sections and to investigate the effect of restrained warping on the mixed torsional problems of FGM sandwich box beams. This beam model is based on the Euler-Bernoulli beam theory for bending and the Vlasov theory for torsion. Material properties of beam are assumed to be graded across the wall thickness. The governing equations are derived from the strain energy expression and the Hermite cubic interpolation polynomials are utilized as shape functions for the finite beam element. Applications are presented for three types of material distributions and three cross-sections with single and double-cells. Through numerical examples, the influences of various structural parameters such as gradient index, thickness ratio of ceramic, material ratio, and width-to-height ratio on the axial, flexural, torsional and warping rigidities of FGM sandwich box beams with single and double-cell sections are parametrically investigated.

The paper is organized as follows: The following section describes a theoretical formulation of the thin-walled FGM sandwich box beam model. The finite element formulation is presented in Section 3. The numerical results and discussions are provided in Section 4. Finally, the article is closed with some concluding remarks.

2. Structural model

2.1. Kinematics and strain energy

Fig. 1 shows the geometry and coordinate systems of a thin-walled FGM sandwich beam with double-cell section. The theoretical development presented in this study requires two sets of coordinate systems which are mutually interrelated as shown in Fig. 2. The first is referred to a Cartesian coordinate system (x, y, z) located at the centroid where the x axis is the member axis and y, z axes are perpendicular to x axis and are not necessary principal inertia axes. The second coordinate system is the orthogonal plate coordinate system (x, s, n) which is defined in the middle contour of the cross-section where the s axis is tangent to the middle surface and is directed along the contour line of cross-section and the n axis is normal to the middle surface of a plate element. The point P is called the pole axis and the angle ψ defines the relative orientation of the (x, y, z) and (x, s, n) coordinate systems, and is equal to the angle between y and s direction at Q .

The present mathematical model of a thin-walled FGM sandwich box beam, the following hypotheses are adopted.

(a) The strains are assumed to be small.
(b) The cross-section contour does not deform in its own plane.
(c) The Kirchhoff-Love assumption is valid for each element. Accordingly, line elements that are normal to the undeformed middle surface remain normal to the deformed middle surface.
(d) Plate forces and moments corresponding to the circumferential stress σ_s and plate shear forces corresponding to the in-thickness stresses σ_{ns} and σ_{xn} are neglected.
(e) The mechanical properties are graded continuously along the wall-thickness according to a prescribed law that is uniform around the contour domain.

According to assumption (a), the plate middle surface displacements $\bar{u}$, $\bar{v}$, and $\bar{w}$ of an arbitrary point in the contour coordinate system can be expressed as follows [16]:

$$\bar{u}(x, s) = U_y \sin\psi - U_z \cos\psi - \theta q \tag{1a}$$

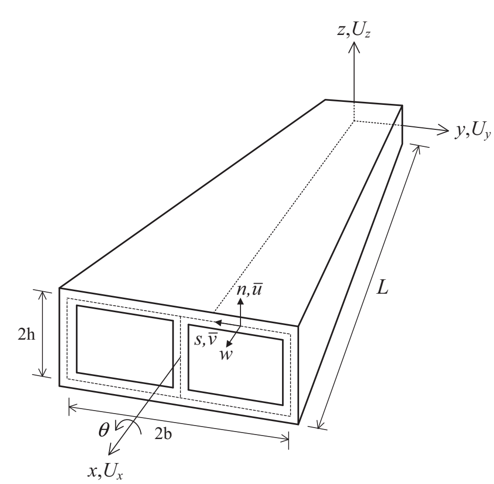

Fig. 1. Geometry and coordinate systems of a FGM sandwich box beam.