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Optimization of axially compressed cylindrical grid structures using analytical and numerical models

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ABSTRACT

The minimum mass designs of state of the art helical circumferential grids are compared to axial helical grids. Analytical models based on the classical laminate theory deliver optimization results in less than one second. Using finite element models the obtained results are validated and adjusted in a second optimization step. Studies on the influence of rib heights and Young's modulus show potential to create lighter grids in axial helical design than in helical circumferential design. This is shown for medium length cylinders from relatively low up to very high load levels.

1. Introduction

Cylindrical grid structures have been used in space engineering since several decades [1]. The main idea is to form a grid of ribs which acts similar to a continuous shell on a global level but has a higher specific bending stiffness.

The present paper deals with weight optimization of axial helical (AH) grids. For comparison optimum results of helical circumferential (HC) grids are presented. Both grid types considered form triangular patterns (see Fig. 1). By shifting or splitting ribs other patterns, like hexagonal, develop. Geometric dimensions and coordinate system of the cylinder are visualized in Fig. 2. Analytical closed-form solutions for analysis and minimum weight of HC grids are given in [1]. Axial stiffnesses of the ribs are smeared to a pseudo layer and the membrane and bending stiffnesses are calculated using the classical laminate theory (CLT). The good agreement between analytical models and detailed finite element (FE) models is documented in [2,3]. In [2] it is also stated that the theoretical advantage of hexagonal HC grids which halve the free length of helical ribs between intersection points and thus increase the local buckling load of the ribs cannot be exploited. It is compensated by the need to increase the rib height to avoid an additional failure mode of buckling of the intersection points in radial direction.

A FE parameter study on hexagonal AH and HC grids with helical angle fixed at 45° can be found in [4]. Applying stability and axial stiffness constraints results in a considerably lighter design for the AH grid. Morozov et al. [5] performed FE parameter studies on hexagonal HC grids under various loading conditions. Similar work but considering different grid types was undertaken by [6]. Although parameter studies give an insight into the influence of certain design variables the maximum potential can only be assessed by considering optimum designs. Also, the restriction of design variables must be handled with care. For example, both of the aforementioned papers assume the same quadratic cross sections for all rib types. This leads to mass penalties compared to an optimized design with independent cross section dimensions.

This paper aims to derive and validate an analytical model for stability and stress analysis of AH grids. With this model AH grids are optimized for minimum mass. Studying the influence of design variables and material properties the potential to find lighter configurations compared to HC grids is shown. FE models are used to validate and adjust the obtained designs in a second stage of optimization. The results are presented for the whole bandwidth of relatively low to very high load levels.

2. Analytical model

It is assumed that the rib spacing is dense enough that the ribs can be smeared to form a continuous pseudo layer. The pseudo layers of different rib orientations are then combined using the assumptions of the CLT. In contrast to a continuous material the ribs only have axial stiffness. The reduced stiffness matrix in the local coordinate system reads

$$Q_{t,11} = \delta_t E_t, \quad t = \mathrm{a},\ \mathrm{c},\ \mathrm{h}. \tag{1}$$

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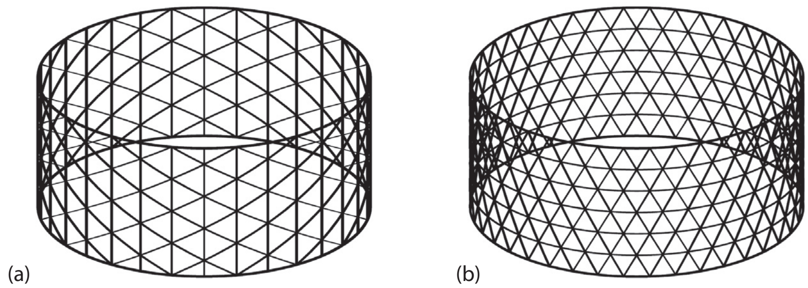

Fig. 1. Isometric view of AH (a) and HC grid (b).

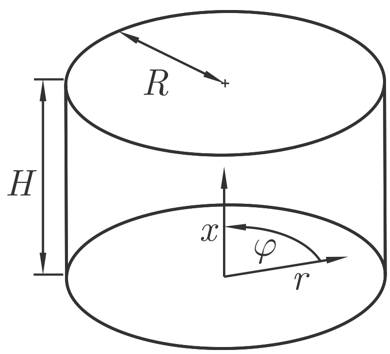

Fig. 2. Cylinder geometry and coordinate system.

Here t stands for rib type (a: axial, c: circumferential, h: helical) and E is the Young's modulus in longitudinal direction. The relations of rib widths to spacings are

$$\delta_t = b_t/a_t, \tag{2}$$

and the relations of rib heights are

$$\beta_t = h_t/h_h. \tag{3}$$

Furthermore, the relations of mass densities read

$$\overline{\rho}_t = \rho_t/\rho_h. \tag{4}$$

In the above equations t again stands for the rib type. If the grid is symmetric to the mid-surface the material law reads

$$\begin{Bmatrix} \underline{N}^0 \\ \underline{M}^0 \end{Bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{D}} \end{bmatrix} \begin{Bmatrix} \underline{\varepsilon}^0 \\ \underline{\kappa}^0 \end{Bmatrix} \tag{5}$$

Using the notation

$$\underline{\underline{\widehat{A}}} = \underline{\underline{A}}^{-1} \tag{6}$$

for the inverse of the membrane stiffness matrix the following non-dimensional stiffness parameters are defined [7]:

$$\varepsilon = \sqrt{\frac{\widehat{A}_{11}D_{11}}{\widehat{A}_{22}D_{22}}}, \tag{7}$$

$$\eta = \frac{D_{12} + 2D_{66}}{\sqrt{D_{11}D_{22}}}, \tag{8}$$

$$\zeta = \frac{2\widehat{A}_{12} + \widehat{A}_{66}}{2\sqrt{\widehat{A}_{11}\widehat{A}_{22}}}. \tag{9}$$

Fig. 3 visualizes the grid geometries and corresponding variables. Taking into account the rib angles $\alpha_a = 0^\circ$, $\alpha_c = 90^\circ$, and $\alpha_h = \pm\alpha$, the membrane stiffness matrix becomes

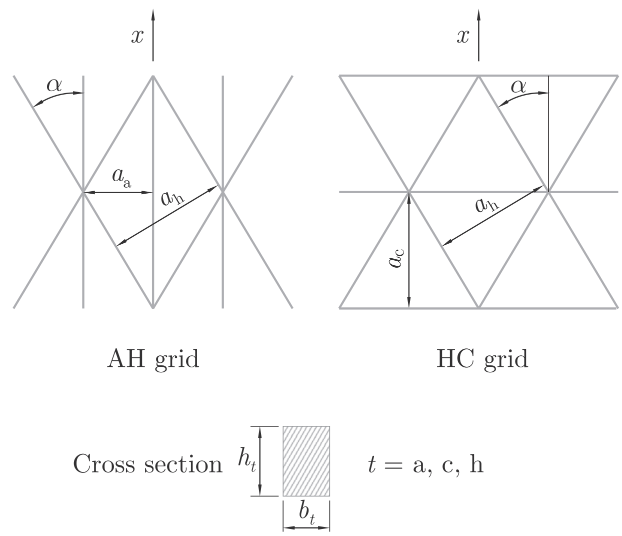

Fig. 3. Grid geometries and corresponding variables.

$$\underline{\underline{A}} = \begin{bmatrix} 2h_h c^4 \delta_h E_h + \beta_a h_h \delta_a E_a & 2h_h c^2 \delta_h E_h s^2 & 0 \\ 2h_h c^2 \delta_h E_h s^2 & 2h_h s^4 \delta_h E_h + \beta_c h_h \delta_c E_c & 0 \\ 0 & 0 & 2h_h s^2 c^2 \delta_h E_h \end{bmatrix}, \tag{10}$$

and the plate stiffness matrix

$$\underline{\underline{D}} = \begin{bmatrix} \frac{1}{6}h_h^3 c^4 \delta_h E_h + \frac{1}{12}\beta_a^3 h_h^3 \delta_a E_a & \frac{1}{6}h_h^3 c^2 \delta_h E_h s^2 & 0 \\ \frac{1}{6}h_h^3 c^2 \delta_h E_h s^2 & \frac{1}{6}h_h^3 s^4 \delta_h E_h + \frac{1}{12}\beta_c^3 h_h^3 \delta_c E_c & 0 \\ 0 & 0 & \frac{1}{6}h_h^3 s^2 c^2 \delta_h E_h \end{bmatrix}. \tag{11}$$

For brevity the trigonometric functions are written as: $s = \sin(\alpha)$, $c = \cos(\alpha)$. As it can be seen from the stiffness matrices, the axial and circumferential ribs only contribute to the longitudinal and circumferential components, $()_{11}$ and $()_{22}$, respectively. In contrast to that, the helical ribs contribute to all populated matrix elements.

An axial compressive load

$$P_x = 2\pi R N_x \tag{12}$$

causes rib stresses

$$\sigma_a = \frac{N_x}{h_h \beta_a \delta_a}, \qquad \sigma_h = 0, \tag{13}$$

in case of an AH grid and

$$\sigma_c = -\frac{s^2 N_x}{h_h c^2 \beta_c \delta_c}, \qquad \sigma_h = \frac{N_x}{2c^2 h_h \delta_h}, \tag{14}$$

in case of a HC grid.