Composite Structures 157 (2016) 222–235

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Vibrational behavior of rotating pre-twisted functionally graded microbeams in thermal environment

Amin Ghorbani Shenas [a], Sima Ziaee [a], Parviz Malekzadeh [b,*]

[a] Department of Mechanical Engineering, Yasouj University, Yasouj 75914-353, Iran
[b] Department of Mechanical Engineering, Persian Gulf University, Bushehr 7516913798, Iran

A R T I C L E I N F O

Article history:
Received 18 June 2016
Revised 13 August 2016
Accepted 28 August 2016
Available online 2 September 2016

Keywords:
Rotating pre-twisted microbeams
Functionally graded materials
Modified strain gradient theory
Free vibration
Thermal environment
Chebyshev–Ritz method

A B S T R A C T

The vibration characteristics of rotating pre-twisted functionally graded (FG) cantilever microbeams are studied. The governing equations are derived based on the modified strain gradient theory (MSGT) of microbeams in conjunction with the first-order shear deformation theory (FSDT) of beams. The material properties are assumed to be temperature-dependent and graded in the thickness direction. In addition to the initial stresses due to centrifugal forces, the initial thermal stresses induced by temperature rise are considered. The Chebyshev–Ritz method is employed to derive the algebraic eigenfrequency equations of the microbeams. After showing the fast rate of convergence and accuracy of the method, the effects of angular velocity, linear and nonlinear variation of the angle of twist along the beam axis, material length scale parameters, temperature rise and material gradient index on the free vibration of pre-twisted microbeams are studied. It is shown that the increase of the free end microbeam twist angle, the nonlinear variation of the twist angle along the microbeam axis, and also the temperature rise reduces the frequencies. On the other hand, by increasing the hub radius, the angular velocity and the length scale parameters, the frequencies increase.

1. Introduction

In recent years, the pre-twisted microbeams have been widely used in micro/nano electromechanical systems (MEMS/NEMS) such as microengines, microturbomachinery and micromachining [1–6], the ultrasonic piezoelectronic motor [7], and the development of a micromotor for *in vivo* medical procedures [8]. In most of these applications, the pre-twisted microbeams undergo a high velocity rotating motion around an axis perpendicular to the microbeam axis. Hence, the study of vibrational behavior of rotating pre-twisted microbeams is important in providing a functional guideline for the accurate design of the above devices.

Functionally graded materials (FGMs), as a new class of materials with physical properties varying smoothly and continuously in the spatial domain, have superior properties with respect to conventional composite materials such as thermal protection from ablation and elimination of stress concentration [9]. This motivates the researchers to produce microbeams made of FGMs, which have high sensitivity and desired performances for different applications in MEMS/NEMS [10,11].

The flexural vibrational behavior of the stationary and rotating pre-twisted beams has been investigated based on the classical beam theories such as Euler-Bernoulli and Timoshenko beam theories by some researchers; see for example Refs. [12–14]. But it is well known that the classical theories (CT) have the deficiency that they cannot characterize the size effect on the mechanical behavior of structures within the micron or submicron scale [15,16].

In recent years, the modified couple stress theory (MCST) and modified strain gradient theory (MSGT) in conjunction with the classical beam theories have been widely used to investigate the effect of size dependence of material properties on the mechanical behavior of microbeams; see for example Refs. [17–35]. In this regards, some researchers have studied the free vibration analysis of rotating microbeams. Mustapha and Zhong [36] presented a mathematical model based on the MCST for the free vibration analysis of a circular doubly-symmetric spinning microbeam embedded in an elastic medium using spectral element method. Dehrouyeh-Semnani [37] presented the flapwise frequency analysis of rotating microbeams connected to a hub by employing the MCST and the FSDT of beams and the finite element method. Ghadiri and Shafiei [38] analyzed the vibrational behavior of the

* Corresponding author.
E-mail addresses: p_malekz@yahoo.com, malekzadeh@pgu.ac.ir (P. Malekzadeh).

http://dx.doi.org/10.1016/j.compstruct.2016.08.031
0263-8223/

rotating FG microbeams based on the MCST and the FSDT of beams under thermal environment. They applied the differential quadrature method to solve the governing equations for cantilever and propped cantilever boundary conditions. Shafiei et al. [39–41] performed extensive research works on the transverse vibration of rotating FG microbeams based on the MCST in conjunction with the Euler–Bernoulli and Timoshenko beam theories. They discretized the governing equations and the related boundary conditions by means of the generalized differential quadrature method (GDQM). Ilkhani and Hosseini-Hashemi [42] obtained the exact solution for the frequencies of the simply supported rotating microbeams by applying the MCST in combination with Euler-Bernoulli and Timoshenko beam theories. They also studied the influences of different parameters on the critical load and type of instability of the microbeams. Dehrouyeh-Semnani et al. [43] studied the flexural frequency characteristics of rotating microcantilevers due to their vibration in the plane of rotating axis by implementation of the MCST and both Euler–Bernoulli and Timoshenko beam theories. They employed the finite element method with two-node beam elements to solve the governing equations of motion subjected to the related boundary conditions.

In the above reviewed important research works, the microbeams without pre-twist angle have been analyzed. To the best of authors' knowledge, the only available research work in the open literature regarding the dynamic analysis of the pre-twisted microbeams is that of Mustapha and Zhong [18]. They derived the coupled elastodynamics governing equation of the twisted microbeams based on the MCST and Euler-Bernoulli beam theory to study the propagating characteristics of a monochromatic bending elastic wave in the microbeams.

It has been shown that the couple stress theory, which is a general form of the MCST, underestimates the size effect because it only employs the rotation gradient and neglects the other gradients [44]. The strain gradient theory (SGT) is the reformulated and extended version of the couple stress theory in which the second-order deformation tensor separated into the stretch gradient tensor and rotation gradient tensor. In order to reduce the number of higher-order material length scale parameters of the SGT, Lam et al. [16] modified this theory and reduced its additional parameters from five to three for isotropic linear elastic materials. This theory can be degenerated into the MCST by appropriate chosen of the length scale parameters.

The micro-structural components made of FGMs are usually under severe of thermal environments during working, which cause the reduction in their stiffness due to softening of the materials, and also, the induced the thermal stresses [23,45,46]. Consequently, their vibrational behavior changes during these operational conditions. Therefore, for a high-quality design and manufacture, these thermal effects should be considered in the mathematical modeling of the corresponding vibrational problems.

To the best of authors' knowledge, the free vibration behavior of the rotating pre-twisted FG microbeams is not studied yet. Hence, as a first endeavor, in this work the influences of angle of twist and angular velocity change on the free vibration behavior of the rotating pre-twisted FG microbeams in thermal environment are studied. The size effect and the global vibrational behavior of the microbeams are accurately modeled by using the MSGT in conjunction with the FSDT of beams. The material distribution of the FG microbeams follows a power law function which determines variation of ceramic and metal fraction along the microbeams thickness. The system of algebraic eigenfrequency equations is derived by using the Hamilton's principle together with the Chebyshev–Ritz method. It should be mentioned that the computational efficiency and accuracy of the Chebyshev–Ritz method for solving complicated structural problems has been established previously [46–48]. After validating the approach, the influences of the non-dimensional angular velocity, linear and nonlinear variation of the twist angle along the beam axis, hub radius, material length scale parameters, material gradient index, microbeam thickness and temperature changes on the non-dimensional frequencies of the microbeams under thermal environment are investigated.

2. Mathematical modeling

The rotating pre-twisted FG microbeams under consideration have the length L, constant thickness h, width b, angular velocity Ω and the angle of twist along the x-axis ϕ (see Fig. 1). The global coordinate axes X-Y-Z are shown at the left-hand end of the microbeam whereas the local coordinate axes x-y-z (in the lower cases), which vary along the microbeam axis as a result of the twist, are shown on the right-hand side. The local x and global X axes are coincident, both passing through the centroid and are perpendicular to the microbeam cross-section, and therefore, represent the axis of twist of the microbeam (see Fig. 2). The beam cross section angle of twist is assumed to vary according to the following rule

$$\phi(x) = \phi_0 \left(\frac{x}{L}\right)^s \tag{1}$$

It is assumed that the microbeams are composed of two different phases (i.e., metal and ceramic phases) and also, the volume

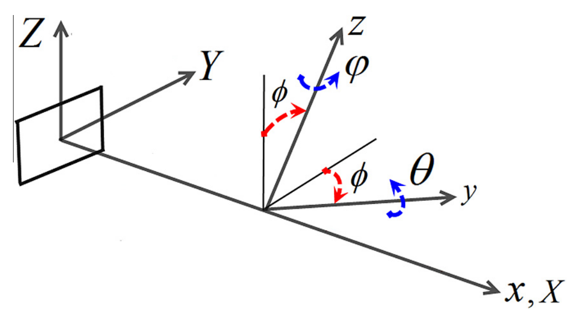

Fig. 2. Th global and local coordinate system together with the rotation components of the rotating pre-twisted microbeams.

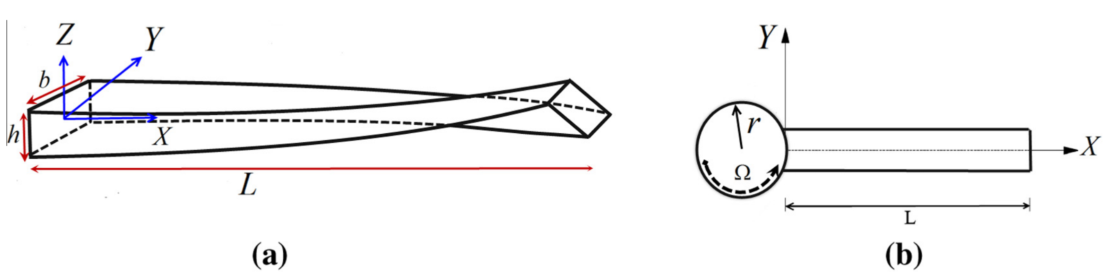

Fig. 1. (a) and (b): Th geometry and global coordinate system of the rotating pre-twisted microbeams.