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A new multi-*p-norm* formulation approach for stress-based topology optimization design

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ABSTRACT

Although the stress-based topology optimization problem has been extensively studied for continuum structures, it is still an open problem and there is still room for improvements. This work proposes a comprehensive approach for dealing with stresses in topology optimization problems. The SIMP method is used to distribute material along the domain. For limiting the stress, a multi-*p-norm* formulation is proposed to deal with the local nature of stress and to avoid stress concentration. This function considers many values of p coefficients at the same time while other formulations adopt a specific value for p defined for subregions. As a consequence this formulation can avoid the stress concentrations without being necessary to define sub-regions. In addition, the proposed formulation is load independent because the multi-*p-norm* is used as the objective function. A SIMP-like formulation is used to address the stress singularity phenomenon and the heaviside projection is used to avoid mesh dependency, checkerboarding, and to control the minimum length-scale. A proper continuation scheme is proposed to all penalization coefficients in order to achieve black-and-white solutions. The optimization problem is solved by using GCMMA. Numerical examples for homogeneous and composite structures are presented to illustrate the proposed formulation.

1. Introduction

A typical problem in Engineering is to design light structures (linear or nonlinear) by limiting their maximum nominal stress, otherwise static fracture or dynamic fatigue failure may occur. An effective way to design these structures is by using structural optimization techniques. Among the available structural optimization techniques, topology optimization gives conceptual design with large mass reduction [1].

From author's knowledge, Duysinx and Bendsoe [2] were the first authors to address topology optimization problems with stress constraints for continuum structures. They have applied the Solid Isotropic Microstructure with Penalization (SIMP) method. However, since this first work, the application of topology optimization problem for stress design is still an open problem. Essentially, there are three well-known difficulties, i.e., the singularity problem, the local constraint problem, and the highly nonlinear behavior of stress constraints [3–12].

The first difficulty is related to the singularity phenomenon which happens when using the SIMP method. Essentially, the singularity behavior of the stress constraints arise when non-disappearing stresses remain as the design variables go towards zero to generate the "void" regions [3,4]. The stress in these regions should be zero because they represent holes, however, these regions can still have a strain which sometimes generates a stress with high value. To solve this problem, many solutions have been proposed in the literature such as ε–relaxation method [2,6,7], the *qp*-relaxation method [13–15], and the relaxed stress indicator method [16].

The second difficulty is related to the fact the stress constraint is a local constraint, thus, essentially, a large number of stress points must be constrained in the domain which generates a high computational cost reducing the efficiency of the optimization solver. To circumvent this problem, some solutions have been proposed in the literature such as the constraint selection method (which considers only active stress constraints) [2,9], global stress measure methods such as the p-norm approach, and the Kreisselmeier–Steinhauser (KS) approach [5,17–19,16,20]. When a global constraint measure is applied, once the effects of the localized stress constraints cannot be considered accurately, is usual to calculate the

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global stress measures in defined subregions of the entire domain [18,19,16,20]. A variation of this method is called clustering techniques, in which essentially stresses for several stress evaluation points are clustered into groups by using a modified p-norm [21]. However, some works proposed solutions to deal with local stress constraints [22] such as an approach based on the augmented Lagrangian technique [23–25]. This approach was extended to multiple loading conditions [24] and contact boundary conditions [25].

Finally, as the third difficulty, the use of a relaxation and a global stress measure methods, and penalization coefficients results in high nonlinear behavior of the stress constraints requesting an efficient and accurate optimizer [2,10,15].

Another important issue in stress based topology optimization problem when using a relaxed model (SIMP or homogeneization) is the post-processing of the results. In this case, the topology optimization results obtained always present some intermediate material which requests some post-processing. However, after the postprocessing, the stress value may increase, mainly in the interface due to the jagged and blurred boundaries. Svärd [26] proposes the IVE (interior value extrapolation) method and shows how to deal with this problem on the interface. Another way to address the problem of jagged boundaries is to use some geometry-based topology optimization techniques, such as level-set method, isogeometric approaches [27,28], and the recent MMC (Moving Morphable Components) method [29,30]. These approaches are based on fixed finite element domains which need to be locally refined at the boundaries to reproduce the boundaries accurately (although the MMC method could be used with mesh-free analysis methods [29]). Thus, re-meshing, spline trimming, or the X-FEM (eXtended FEM) can be used with these approaches. However, these approaches are beyond the scope of this work.

Stress constrained topology optimization problems have also been solved by using the level set method [31–39]. In this method, the singularity problem does not occur because the stress constraints are considered only for the solid phase. In addition, there are no intermediate values of design variables between solid and void. Most part of works consider local stress constraints [33,36–39], however, due to the large number of design variables and stress constraints, some works employ global stress measures to reduce the computational cost [31,32,34–36]. While some works have not addressed the jagged boundaries issue [31,33], others have applied some re-meshing techniques [34,38,39], the X-FEM [40,32,35,36], and the FCM (Finite Cell Method) [37] to model the boundaries more accurately and avoid jagged interfaces. Finally, the level set has also been combined with the phase field method [22,39]. However, these methods essentially can only join holes and cannot create them, thus, their solution is highly dependent on the initial guess which limits their design space in relation to SIMP methods [33,15]. To try to circumvent this problem they are combined with topological derivatives [41,33,39] which provides an appropriate initial guess. Although, topological derivatives itself have provided satisfactory results when solving local stress constraint problems with different failure criteria [42,43], they can be defined, and thus it is limited, only for linear problems. In this sense the use of topological derivatives to provide an initial guess for other methods in linear problems could be reconsidered, once the topological derivative itself provides a solution [42,43]. Besides, it is well known that level set methods are very complex to implement and present some unfavorable numerical issues [44].

Thus, these previous works show that despite of its problems, the SIMP method for solving stress constrained topology optimization problems still has advantages and studies to improve it are valid. Thus, trying to contribute to this field, in this work, we are proposing a stress-based topology optimization formulation which uses a global measure stress formulation based on a defined multi-*p-norm* function. This function considers many values of p coefficients at the same time while other formulations adopt a specific value for p. As a consequence this formulation can avoid the stress concentrations without being necessary to define sub-regions such as in the works of Le et al. [16] and Holmberg et al. [21], which is a new concept. These sub-regions are dynamics once they must be redefined as the stress distribution changes. This generates numerical instabilities in the problem because they make the process discontinuous between iterations. Other point is that when the multi-*p-norm* stress function is considered as a constraint, it is necessary a multiplying coefficient, once the *p-norm* does not give the exact maximum value. In Le et al. [16], this multiplier is not differentiable, and in Holmberg et al. [21], this multiplier does not estimate precisely the maximum. Thus, in this work, the multi-*p-norm* is considered as the objective function which besides eliminating this multiplier, reduces the sensitivity in relation to the applied load, and thus, giving a more generic result. Additionally, the penalization coefficient values for the stiffness and stress calculations usually applied in previous works [16,21,39] are not able to achieve good black-and-white solutions, as demonstrated in Section 5.1. Thus, we propose a different penalization scheme together with the heaviside projection technique [45] to improve the results.

We apply the method for homogeneous and composite structures (combination of different homogeneous materials). The finite element model is based on plane and shell elements and the material model is based on SIMP [2]. To solve the optimization problem, we apply the iterative optimization algorithm GCMMA (Globally Convergent Method of Moving Asymptotes) [46,47]. As a result, stress-based topology optimization designs of homogeneous and composite structures are presented.

This paper is organized as follows: the stress-based topology optimization problem formulation is described in Section 2; the definition of the multi-p-norm is described in Section 3; the sensitivity analysis is presented in Section 4; numerical examples are presented in Section 5; finally, some conclusions are inferred in Section 6.

2. Stress-based topology optimization

It is well known that, in stress-based topology optimization methods that adopt density methods (such as the SIMP) as the material model, the stresses need to be relaxed in order to avoid the singularity problem caused by degenerated subspaces [6,2]. Besides that, the optimization formulation must be able to avoid stress concentration that can occur at sharp interfaces, such as reentrant corners. Additionally, a filtering technique must be used to avoid common problems in topology optimization, such as mesh dependency, checkerboarding, and narrow members. In this section, all these formulations are described, together with the proposed optimization formulation.

2.1. Finite element method

In the topology optimization method (TOM), FEM is also applied to discretize the material distribution, where each element is associated to a pseudo-density: a parameter that dictates whether the element is solid or void. The examples presented in this work are optimized by assuming linear elasticity and static behavior. Thus, the equilibrium equation can be written as:

$$\mathbf{KU} = \mathbf{F} \quad (1)$$

where $\mathbf{K}$ is the global stiffness matrix of the structure, $\mathbf{U}$ is the global nodal displacement vector, and $\mathbf{F}$ is the external load vector. $\mathbf{K}$ can be properly built by using the stiffness matrices of the elements, given by

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