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A review on plate and shell theories for laminated and sandwich structures highlighting the Finite Element Method

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This paper is dedicated to Professor J.N. Reddy.

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ABSTRACT

In engineering, the amount of complex geometrical problems, which needs to be solved by using plates and shells theories, is remarkable. This is the reason why there are so many theories, which attempt to simplify three-dimensional problems into simpler ones. Additionally, the current increasing use of laminated and sandwich structures demands a minimum of accuracy from two-dimensional formulations. In the literature, one can find a variety of bi-dimensional theories and solution methods to solve these problems. Laminated and sandwich structure formulations are mainly classified according to the treatment of the variables in the normal direction of the plate/shell surface: equivalent single layer, Zig-Zag and layer-wise theories. The contribution of this paper is to set the stage for new theories and solution methods for laminated and sandwich structures by reviewing over 100 papers. To show the importance of the coupling between plate/shell theories and the respective solution method, a detailed review on theories and the respective solution methods is firstly given to update the current state of art. After that, solution methods based on the Finite Element Method are explained to exhibit how particular and/or complex an approach can get. In fact, this review gives a clearer picture on plate/shell theories.

1. Introduction

Plate and shell theories consist in an old subject. Professor J.N. Reddy has significantly contributed in the development of theories and solution methods for laminated plates, shells and sandwich structures throughout his academic career. Some of his extensive work can be exemplified by some published books and papers [1–5]. However, many other researchers also study the mechanics of plates and shells, because new materials and complex structures have been created every day to attend the requirements of the top-notch engineering applications. Therefore, reviews on plates and shells theories and solution methods are constantly required.

To achieve an accurate structural solution, there are innumerous variables, which need to be handled. Depending on the loading, geometry or constitutive complexity of the problem, very particular approaches might be required. Laminated plates and shells and sandwich structures present an outstanding complexity regarding its microscopic and macroscopic behavior [6]. Intelligent structures may be regarded as a class of laminated or sandwich structures. Such structures can be employed as sensors and/or actuators, depending whether the structure was designed to monitor (passive) or to actuate (active) in the structure [7–9].

Wind turbine blades design is another current application of laminated and sandwich structures [10]. For instance, the use of sandwich structures in this application helps one to reduce the weight of this structure but, usually, it complicates the designing of such structures [11,12].

Whichever the application, there must be a proper theory and a solution method to solve the problem at hands. However, the answer to which theory/method to be used is becoming blurrier with each new approach. That is why; this paper reviews not only plate theories, but also the respective solution methods. The coupling, which exists between theories and solution methods, becomes clearer with this review.

This review is split into two sections, both chronological. In the first section, a detailed review on plate/shell theories is provided along with comments on the respective solution methods, whenever applicable. Examples of asymptotic, axiomatic and continuum based formulations for plates and shells can be found in this section. Due to the simplifications assumed during the derivation process, the references show that a coupling between the theory and

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the respective solution method is always present. Because of this, the accuracy of such theories becomes dependent on the solution method. This is clearly seen in the second section, where the review is focused (but not restricted to) on the plate/shell theories solved by the Finite Element Method (FEM). This review was dedicated to this method due to its popularity, which is grounded on the applicability of this method in engineering.

This paper finishes with a discussion on the development and use of the so-called Unified Formulations (UFs) for plates and shells.

2. Review on plate/shell theories

This review begins with the work of Srinivas and Rao [13]. They worked on an exact solution for bending, vibration and buckling problems of thick orthotropic rectangular plates and laminates. The method for deriving exact solutions was based on double Fourier series. Kirchhoff's thin plate theory and Reissner–Mindlin's thick plate theories for small strains were investigated. Results showed a dependence of the laminate orthotropic properties and the assumption of thin or thick laminate. Similarly, Kulkarni and Pagano [14] proposed an approach to solve the dynamics of composite laminates in cylindrical bending by using Mindlin's plate theory, as well. In the displacement formulation, a time dependent exponential term multiplies the single Fourier series. Analytical dispersion results for symmetrical and unsymmetrical laminates were discussed by the aid of non-dimensionalized frequency and phase velocity values. For low anisotropy and symmetrical laminates, the proposed approach showed the best results.

Di Sciuva [15] developed an equivalent single layer anisotropic plate element based on Ambartsumian's ZZ (Zig-Zag) theory. Even though the author refers to his formulation as discrete, it is not a layer-wise theory, because the number of degrees of freedom of the element does not depend on the number of layers. It is a 4-node (with 8 DOFs – degree of freedoms per node) rectangular element, including bending, extension and transverse shear contributions. Since it treats shear strains as nodal parameters, it is defined as a mixed approach with eight variables per node (three displacements, three rotations and two shear strains). Hermitian polynomials were used as interpolation functions for the transverse displacement fields and the remaining variables linearly. The idea was to preserve C-1 continuity requirements for the bending problem. A first order shear deformation theory was chosen to estimate the displacement fields. Cylindrical bending of cross-ply and angle-ply laminates and bending of a square cross-ply laminate were investigated via this element. Good agreement was observed in both low and high span-to-thickness ratios.

An improved plate formulation was derived by Toledano and Murakami [16] based on the Reissner's mixed variational approach [17]. The Zig-Zag approach using Legendre polynomials from the previous work of Murakami [18] was developed. However, higher order displacement functions were chosen, more specifically, they assumed displacements from Lo et al. [19]. The resultant laminate formulation fitted in the equivalent single layer type of laminated plate theory. Once again, the cylindrical bending of symmetrical and non-symmetrical cross-ply laminates was investigated. Macroscopic responses of this formulation, such as central deflection, were accurately modeled when compared to a reference exact solution. The results retained a high level of accuracy even at small span-to-thickness ratios. Comparison of Murakami's first order shear deformation Zig-Zag theory to the non-Zig-Zag high order formulation of Lo, Christensen and Wu indicated that the previous one is more accurate in modeling laminated plates.

Owen and Li [20,21] presented a three dimensional first order shear deformation plate element in two companion papers. The solution method exploited a sub-structuring technique, which allows one to increase the actual number of layers in the laminate. However, the main advantage of this sub-structuring was the fact that only one set of layer equations needed to be solved. All the other layer equations could be solved by using the results of this one set of equations, which had been solved. This dramatically reduces the size of the problem. However, an increase in the total processing time is expected. For a better comparison to elasticity solutions, a smoothing technique was used to interpolate the results from the integration points to the boundaries of the element. This is particularly appealing for transverse stresses with selective or reduce integration. Thin and thick laminated composite plates in static, dynamic and stability problems were investigated. Since it is a layer-wise type of plate formulation, good accuracy was obtained for displacements and stresses across the thickness of the laminate by the linear approximation. Nonetheless, a comparison using 27 nodes for a element with higher order variation of the in-plane displacement was performed. It did not show considerable improvement of the results in order to justify the increase of parameters.

A higher-order theory satisfying the zero transverse shear stress on the top and bottom layers of laminated plates was presented by Kant and Pandya [22]. A C-0 nine-node iso-parametric quadrilateral element is derived. The laminated theory was an equivalent single layer type. Seven degrees of freedom were determined per node. Three translations and two rotations plus other two special rotations were considered. These special rotation DOFs were used to model two transverse shear strains. Since continuity is not guaranteed when calculating the stress within each layer via constitutive relations, integration of equilibrium equations was chosen as a more accurate technique. It requires extra post-processing of results, but the level of accuracy is much higher. However, the presented results were still not exact according to the references consulted in the article. The model also performed well from the limit of thick plates (Length or width per thickness $\geqslant 10$) without shear correction factors. More evaluations of this formulation can be found in Pandya and Kant [23].

In 1990, Professor J.N. Reddy revised a variety of the current third-order plate theories [1]. Differences and similarities between such theories were pointed to identify the actual contributions. After compiling all of these third-order theories, Reddy proposed a general, consistent-strain plate theory from which any of the reviewed theories can be derived. He also accounted for non-linear strains by using von Kármán's assumptions. No validations or comparisons were conducted.

To determine static and dynamic characteristics of plates and shells, Noor et al. [24] investigated a two-step solution procedure. Based on a predictor first step, an overall gross response of the plate was obtained through a first order shear deformation theory by using the aid of shear correction factors. Then, in a second step, a correction of the predicted values was carried out by two different methods. The first one estimated the shear correction factor to calculate the stresses, while the second method used the dependence of the displacement components on the thickness coordinate. Based on the simulated tests, the two-step procedure was sensitive to the symmetry of the laminate as well as on the shear correction factors. One of the main advantages of the method is the easy and cheap implementation of the predictor phase. Hence, via post-processing (correction), better results can be achieved without the need of complicated element formulations.

Touratier [25] proposed a new type of expansion of the thickness coordinate to derive a plate formulation. Instead of adding one more non-linear term to the expansion of the thickness coordinate in the in-plane displacement assumed functions [26], a sine function was chosen. This is endorsed with the fact that the derivative of the sine function is the cosine function, which is an even