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## Geometrically nonlinear analysis of sandwich structures

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## ABSTRACT

In this work a finite element model is extended for geometrically nonlinear analysis of sandwich plate–shell structures, to study the nonlinear static response of sandwich plates or curved shells which can have a hard or soft core sandwiched between stiff elastic layers. The finite element is obtained by assembling all element–layers through the thickness using specific assumptions on the displacement continuity at the interfaces between layers, but allowing for different behavior of the layers. The stiff elastic layers are modeled using the classic plate theory and the core is modeled using the Reddy's third order shear deformation theory. Using the Newton–Raphson incremental–iterative method, the equilibrium path is obtained, and in case of snap-through occurrence the automatic arc-length method is used to track the full load displacement distribution. This simple and fast element model is a non-conforming triangular flat plate/shell element with 24 degrees of freedom for the generalized displacements. It is benchmarked in the solution of some illustrative plate–shell examples and the results are presented and discussed with numerical and experimental alternative models.

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## 1. Introduction

Sandwich structures, generally consist of two stiff face sheets and a soft core, which are bonded together, exhibiting many valuable properties, and for this reason are used in engineering practice, such as aerospace, automobile, and shipbuilding. The most important advantages of sandwich structures are for example, good thermal and acoustic isolation, good vibration damping, high strength and stiffness, low weight and durability.

Many works in nonlinear analysis for general composite structures, using the finite element method have appeared in the last decades. Some works were carried out specifically in geometrically nonlinear analysis of sandwich plate structures. However, there are very few works in those studies concerning sandwich shell structures with very soft core. In these sandwich plate/shells structures with a relatively soft core compared with the stiff face sheets, the deformation behavior is essentially different from the corresponding behavior of classical Equivalent Single Layer (ESL) structures. To account for the geometrically nonlinear behavior, specific sandwich plate and shell models have been developed. Noor et al. [1]

and Carrera and Brischetto [2] published extensive reviews on computational models for sandwich panels and shells in a wide range of fields, where very few works are mentioned on the nonlinear analysis of soft core sandwich plates and shells. Carrera and Brischetto [3] carried out a study where a variety of classical and advanced shell theories are assessed to evaluate bending response of sandwich shell structures with soft core. Frostig et al. [4] developed a nonlinear Higher-Order Shear Deformation Theory (HSDT) for the bending behavior of a sandwich beam with a core that is transversely flexible. The formulation uses a beam theory for the skins and a two-dimensional elasticity theory for the core. The behavior is presented in terms of internal resultants and displacements in the skins, peeling and shear stresses in the skin–core interfaces, and stress and displacement fields in the core. Bausen et al. [5] conducted an experimental set-up for the analysis of large deflections of clamped sandwich plates, and compared the results with a finite element analysis. Carrera [6] presented a nonlinear finite element formulation applied to sandwich plates and based on a zig-zag distribution of the in-plane displacements in the thickness direction, fulfilling interlaminar equilibrium for the transverse shear stress components. The model was compared with 3-D solutions and other (HSDTs) for linear and non-linear bending, free vibration behavior, in-plane buckling and post

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buckling problems. Hao et al. [7] examined buckling and post buckling behavior of sandwich plates using the finite element method where the face sheets and the core are treated separately as three-dimensional solids. Librescu et al. [8] developed a finite element model for linear and nonlinear static response of curved sandwich shells. The analysis is based on a geometrically nonlinear theory of shallow sandwich shells restricted to the cases of small strains and moderately small rotations. Ferreira et al. [9] carried out a non-linear formulation for general arbitrary sandwich shells, accounting for geometric and material non-linear behavior using the Ahmad shell element [10]. Hohe and Librescu [11] presented a geometrically nonlinear shell theory for doubly curved structural sandwich panels using a model based on the adoption of the Kirchhoff theory for the face sheets and a second/third order power series expansion for the core displacements. Marcinowski [12] proposed a finite element model based on the Ahmad original element and on the author's adaptation for the nonlinear analysis of sandwich plates and shells. It was assumed that in the faces and in the core the materials exhibit orthotropic properties and the structures are composed of a thick and soft core, covered on both sides by very thin and stiff faces with the ratio of thicknesses greater than 15. Nonlinear analysis for a sandwich shell panel was carried out using the developed model and the results compared with the ones obtained by a solid 3D model and a shell model, both available on Program COSMOS/M [12,13]. Sokolinsky et al. [14] obtained experimental results of four-point bending tests of sandwich beams with aluminum face sheets and Divinycell (PVC) foam core. The results are compared with the ones of Frostiğs [4] HSDT model. Moreira et al. [15] established an approach to the layerwise formulation for composite structures, relying on the use of solid-shell finite elements, based on Enhanced Assumed Strain (EAS) methodology. The four and eight node finite element models developed can be applied to material and geometrically nonlinear analysis of soft core sandwich structures. Madhukara and Singha [16] developed a four node shear flexible rectangular plate bending element based on von Kármán's assumptions and using the ESL theory, to study the geometrically nonlinear static and dynamic behavior of sandwich plates made with soft core. The finite element model is developed based on higher order displacement fields, incorporating transverse shear and normal deformation. Linear polynomial shape functions are employed to describe the in-plane displacement field variables and the transverse strains,  $u_0, v_0, \gamma_{xz}^0, \gamma_{yz}^0$ , respectively, and cubic polynomial functions are considered for out-of-plane generalized displacements  $w, w^*$ .

From the literature review, we conclude that finite element models based on simple facet elements and using the ESL theory are usually good for the analysis of common plate-shell structures. However, these models may not be appropriate for the analysis of sandwich structures, including geometrically nonlinear analysis, depending on the ratio of the face-to-core stiffness and the ratio of the face-to-core thickness.

A finite element model based on [17,18] is extended to carry out the geometrically nonlinear static analysis of sandwich plate-shell structures considering cases where snap-through occurrence exist and to study the corresponding load displacement paths. This model is applied to sandwich plate-shell structures with stiff and soft cores, and the predicted results are compared and discussed with those of alternative numerical models and with experimental results.

## 2. Sandwich plate model

### 2.1. Displacement field

In Fig. 1 the geometry of a typical sandwich plate is shown.

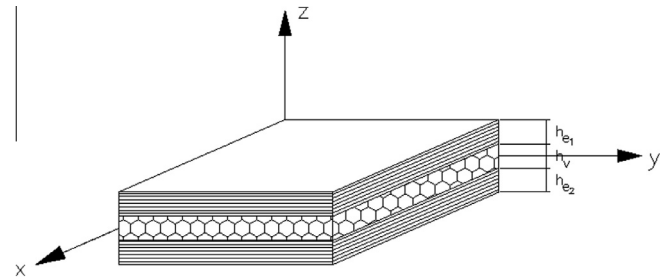


Fig. 1. Sandwich plate.

The finite element model is obtained by the assembly of  $N$  "elements" through the thickness and where the displacement continuity at the interfaces between layers is imposed [17]. For the soft layer, Reddy's third-order shear deformation theory [19] is assumed. Thus, the displacement field is:

$$\begin{aligned} u(x, y, z) &= u_0^c(x, y) - z\theta_y(x, y) + z^3c_1 \left[ \theta_y(x, y) - \frac{\partial w_0}{\partial x} \right] \\ v(x, y, z) &= v_0^c(x, y) + z\theta_x(x, y) + z^3c_1 \left[ -\theta_x(x, y) - \frac{\partial w_0}{\partial y} \right] \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (1)$$

where  $u_0^c, v_0^c, w_0$  are displacements of a generic point in the middle plane of the core layer referred to the local axes  $-x, y, z$  directions,  $\theta_x, \theta_y$  are the rotations of the normal to the middle plane, about the  $x$  axis (clockwise) and  $y$  axis (anticlockwise),  $\partial w_0/\partial x, \partial w_0/\partial y$  are the slopes of the tangents of the deformed mid-surface in  $x, y$  directions, and  $c_1 = 4/3h^2$ , with  $h$  denoting the total thickness of the structure.

For the stiff face sheet elastic layers the Kirchhoff-Love theory is assumed. The corresponding displacement field is [17]:

$$\begin{aligned} u^i(x, y, z) &= u_0^i(x, y) - (z - z_i) \frac{\partial w_0}{\partial x} \\ v^i(x, y, z) &= v_0^i(x, y) + (z - z_i) \left( -\frac{\partial w_0}{\partial y} \right) \\ w^i(x, y, z) &= w_0(x, y) \end{aligned} \quad (2)$$

where  $u_0^i, v_0^i$  are the in-plane displacements of a generic point in the middle plane of the  $i$  layer,  $\partial w_0/\partial x, -\partial w_0/\partial y$  are the slopes of the tangents of the deformed mid-surface in  $x, y$  directions respectively, and  $z_i$  is the  $z$  coordinate of the mid-plane of each layer, with reference to the core layer mid-plane, and  $i = e_1, e_2$  is the index, of upper and lower stiff elastic layers respectively.

In this formulation, displacement continuity between layers is considered. Thus, the displacement field in any layer can be obtained from the displacement field of the soft core layer, taking in consideration the conditions of kinematic links [17].

$$\begin{aligned} u^{(e_1, e_2)}(x, y, z, t) &= u_0^v - z \frac{\partial w_0}{\partial x} (\mp) C \frac{\partial w_0}{\partial x} (\pm) C \theta_y \\ v^{(e_1, e_2)}(x, y, z, t) &= v_0^v - z \frac{\partial w_0}{\partial y} (\mp) C \frac{\partial w_0}{\partial y} (\mp) C \theta_x \end{aligned} \quad (3)$$

where  $C = -(h_c/2) + c_1(h_c/2)^3$ . Hence, the number of generalized displacements is reduced to 7, namely  $u_0^c, v_0^c, w_0, -\partial w_0/\partial y, \partial w_0/\partial x, \theta_x, \theta_y$ .

### 2.2. Linear and nonlinear strains

The present model considers large displacements, large rotations and small strains. The Green's strains components associated

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