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A novel mixed-mode cohesive formulation for crack growth analysis

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ABSTRACT

Modeling fracture with a cohesive zone model requires an appropriate cohesive law for correlating interfacial tractions and crack face separation, especially in mixed-mode loading scenarios. Various approaches have been employed in order to develop such a law which can be characterized into potential-based and non-potential-based formulations. A critical re-examination of these methods is presented here, followed by a novel mixed-mode formulation which satisfies a physical criterion for crack propagation. Specifically, as the crack propagates, the trailing current crack tip is defined through the vanishing of normal and tangential components of the interfacial tractions simultaneously. A general formulation for mixed-mode conditions is proposed in this paper. In particular, given normal and tangential traction separation laws for pure normal and tangential modes as being material properties, the normal and tangential traction separation laws in mixed-mode loading are formulated so that all traction components disappear at the same effective separation when the crack advances. The new mixed-mode law is used to analyze three standard fracture problems in laminated composites, including double cantilever beam (DCB), end-notch flexure (ENF), and mixed-mode bending (MMB). A comparison with predictions from some selected mixed-mode cohesive laws and experimental data available in the literature is also included to further validate the proposed mixed-mode law.

1. Introduction

The finite element method is a mainstay in many continuum mechanics problems and its utility for analyzing fracture initiation and growth problems is aided by cohesive zone models, initially advanced by Barenblatt and Dugdale [1,2]. In such an analysis, an appropriate cohesive law that is capable of capturing different failure modes (Normal/Mode-I and Tangential/Mode-II) as well as their interactions under mixed-mode scenarios is required to relate interfacial traction and crack opening. In the literature, approaches proposed for the development of this law can be classified into two main categories: potential-based [3–9] and non-potential-based formulations [10–15].

For the potential-based approach, the mixed-mode traction-separation laws are derived from a potential function. Normal and tangential traction-separation laws are the derivatives of this potential function with respect to the corresponding normal and tangential separations. Thus, each traction component is a function of both normal and tangential displacements. These traction components have to reduce to the corresponding normal and tangential traction separation laws in pure mode situations. Hence, the potential function has to be selected so that this condition is satisfied. In the work of [3,4], this potential function is constructed using a one-dimensional generalized relationship between the equivalent traction and separation. With this approach, crack propagation occurs when the equivalent separation reaches a critical value. The derived normal and tangential traction-separation laws simultaneously vanish at the same combination of normal and tangential separations without additional boundary condition (criterion) to force the traction to drop to zero at some specific crack displacement. This means that no additional enforcement criterion is required for achieving the physical consideration of crack propagation. However, in order to satisfy the requirement that mixed-mode cohesive laws reduce to the corresponding traction-separation laws under pure-mode loading, the fracture toughness of pure normal and tangential modes are chosen to be the same. This is non-physical and is unsupported by experimental data, [16]. Instead of formulating a potential function based on a generalized traction-separation relationship with an effective separation, [5–9] proposed several forms for the potential function in terms of normal and tangential separations. Crack propagation was treated by additional conditions imposed at the final crack opening. For instance, this was demonstrated in one of the most recent potential based models, proposed in [9] and referred to as the PPR model, by defining conjugate quantities for the final

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normal and tangential crack openings and using these four values to specify two cohesive interaction regions for normal and tangential modes. Each cohesive traction is set to zero if the separations are outside of its defined cohesive interaction regions [9]. This indicates that one traction component may become zero while the other traction component has not yet vanished, especially at the boundary of the smaller defined cohesive interaction region.

For non-potential-based models, normal and tangential traction separation laws under mixed-mode loading are not required to be related through a potential function. [10,11] utilized a model in which traction separation laws for Mode-I and Mode-II under mixed-mode scenarios take the corresponding forms in pure Mode-I and pure Mode-II. Thus, the normal traction separation law is a function of only the normal separation while the tangential traction-separation law is a function of only the tangential separation. Their interaction in mixed-mode loading is governed through extra criteria such as the widely used power law criterion for crack growth [10,11]. The crack propagation is treated through an enforcement of tractions to be zero when the criterion is met. Rather than tracking the normal and tangential traction separation laws independently until the crack growth criterion is met as in [10,11], [12] introduced a single relative displacement and constructed a single effective law for following the mixed-mode interaction. The relative displacements at the softening onset and the propagation in the effective law are determined from specified initiation and propagation criteria, respectively. These critical values are dependent on a mode mixity ratio β between the normal and tangential separations. This leads to the dependence of the effective law on the ratio β. Thus, though only a single relative displacement parameter is employed to keep track of the normal and tangential tractions under mixed-mode loading, the effective law is not fixed (evolving) if the ratio β is unfixed (changing). In such a case, there is no guarantee that enforcement of traction to zero smoothly (without a finite jump) does occur at the transition to crack propagation. If the ratio β is fixed, such as at the softening onset, a fixed single effective law is constructed. However, this fixed single effective law might not be able to physically capture the responses of problems where the ratio between the normal and shear separation keeps changing. For such problems, the use of the fixed effective law also leads to the enforcement of traction to zero (through a jump) at the transition to crack propagation. In addition to these non-potential-based models, cohesive laws in which each traction is a function of both normal and tangential separations are also proposed [13–15]. These models utilized a fixed single effective law in order to develop the mixed-mode cohesive laws. This aspect resembles an approach used in the potential-based methods of [3,4]. Hence, no extra criterion or boundary condition has to be imposed to force tractions to drop to zero when a crack propagates. In addition, in order to ensure that the model can capture different fracture energies of different loading modes, a scaling factor is introduced. In [13–15], this factor is chosen to be a non-dimensional constant. Nevertheless, the use of a single effective law and a constant scaling factor restricts these models to work with cases where pure normal and tangential traction separation laws take the same forms, such as cubic polynomial form in [13] and bilinear form in [14,15]. A more general approach that can combine any given form of traction-separation laws in pure Mode-I and pure Mode-II to construct cohesive laws for mixed mode loading is, therefore, proposed. Specifically, inputs of this proposed model are the traction-separation laws in pure mode scenarios with no assumption on their forms or shapes. The advantage of the models in [13–15], which is that crack propagation is physically captured in the formulation of the mixed-mode laws with no extra enforcement of traction jumps to zero, is still preserved.

Though various models, as discussed above and also summarized in a recent review [17], have been presented for cohesive zone modeling, there is relatively little work that is concerned with how different the physical consideration of crack propagation is treated in these models. Therefore, there is a need to re-examine how different models satisfy the requirement of vanishing tractions at crack propagation in order to better understand the similarities and differences amongst the current models with respect to this physical requirement. Therefore, Section 2 of this paper focuses on providing a more detailed discussion on this issue. It also reveals the need for a general extension of cohesive zone modeling for mixed-mode loading which can incorporate different forms of given pure normal and tangential modes while still not requiring any additional enforcement to achieve the physical requirement that tractions mush vanish simultaneously, and smoothly, as the crack propagates. Such a general mixed-mode law is discussed in Section 3. Additionally, since quantitative comparisons of the predictions obtained from various mixed-mode models were normally not presented systematically, Section 4 presents a comparison between predictions obtained from some selected potential-based and non-potential-based models for standard fracture problems such as DCB, ENF, and MMB. A comparison with experimental data available in the literature [16] is also included to further study the predictive capability of different mixed-mode laws. Discussions and conclusions are included at the end of this final section.

2. Review of crack propagation in mixed-mode fracture models

2.1. Potential-based mixed-mode formulation

2.1.1. Potential-based mixed-mode formulation with correlation to a one-dimensional generalized traction-separation law

A one-dimensional generalized traction-separation law provides a relationship between the equivalent traction (T) and the equivalent displacement (δ). Depending on the considered problems, several forms, such as cubic polynomial, trapezoidal, exponential, linear, and bilinear functions [17], can be employed for the construction of this generalized law. For example, in the study of Tvergaard and Hutchinson [3], a trapezoidal relation, as shown in Fig. 1, is used in which a non-dimensional effective separation λ is utilized as the equivalent displacement. This effective separation is defined as follows:

$$\lambda = \sqrt{\left(\frac{\delta_n}{\delta_n^*}\right)^2 + \left(\frac{\delta_t}{\delta_t^*}\right)^2}, \tag{1}$$

where δ_n^*, δ_t^* are the final crack openings in pure normal and tangential modes while δ_n, δ_t are the normal and tangential separations, respectively. A potential function, $\Phi(\delta_n, \delta_t)$, is then constructed as:

$$\Phi(\delta_n, \delta_t) = \delta_n^* \int_0^{\lambda} T(\bar{\lambda}) d\bar{\lambda}. \tag{2}$$

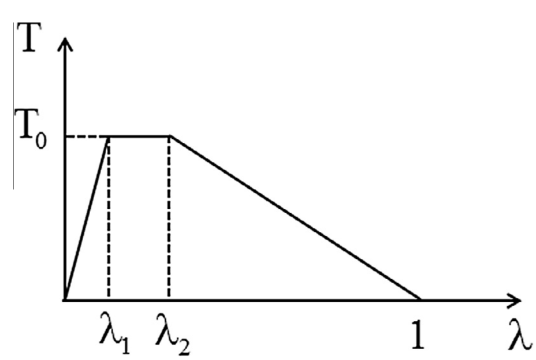

Fig. 1. Trapezoidal generalized traction separation law.

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