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A general semi-analytical solution for consolidation around an expanded cylindrical and spherical cavity in modified Cam Clay

Hang Zhou [a,b,*], Hanlong Liu [a], Yahui Zha [a], Feng Yin [a]

[a] Key Laboratory of New Technology for Construction of Cities in Mountain Area, College of Civil Engineering, Chongqing University, Chongqing 400045, China
[b] State Key Laboratory for GeoMechanics and Deep Underground Engineering, China University of Mining & Technology, China

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ABSTRACT

This paper presents a general semi-analytical solution for undrained cylindrical and spherical cavity expansion in Modified Cam Clay (MCC) and subsequent consolidation. The undrained cylindrical and spherical cavity expansion response in MCC model is obtained through the similarity solution technique. Then, the subsequent consolidation process around the cavity is governed by the classical partial differential equation for consolidation. Finite Difference Method (FDM) is selected for solving the consolidation equation numerically. The proposed semi-analytical solution is validated by comparing the prediction of the dissipations of the pore pressure with Randolph's closed-form solution for elastic-perfectly plastic soil. Parametric study shows that G_0/p'_0, R and M have significant influence on the cavity wall excess pore pressure dissipation curve, while it is not sensitive to the value of v'. It is also found that the negative pore pressure generates around the expanded cylindrical and spherical cavity wall during the consolidation process when $R > 5$ for typical Boston blue clay. The developed solution has potential applications in geotechnical problems, such as the pile foundation, in-situ test, tunnel construction, compaction grouting, and so forth.

1. Introduction

Consolidation around an expanded or contracting spherical or cylindrical cavity is a classical boundary value problem in geomechanics. Since this mechanical model has numerous applications in geotechnical engineering, such as pile foundation, in-situ test (CPT test, pressuremeter test), tunnel construction, compaction grouting, it receives extensive concerns and various analytical, semi-analytical and numerical solutions have been presented in the previous. The earliest work is contributed by Cryer [4], who proposed an analytical solution for a poroelastic sphere subjected to an external radial stress field in the presence of the boundary drainage. In order to investigate the dissipation of the excess pore pressure induced by the pile driving, Randolph and Wroth [9] derived a closed-form solution for consolidation around a driven pile (expanded cylindrical cavity), assuming that the soil skeleton deforms elastically. Later, Carter [1] developed a semi-analytical solution for the radial dissipation of pore water pressure around a freshly created, vertical hole and used this solution to capture the swelling effect around the borehole. Then, Scott [10] studied the problem of radial consolidation around the radial compression of a cylinder and a sphere, and expanded cylindrical cavity in phase-change soil. Subsequently, Zhou et al. [12] presented an analytical solution for investigating the coupled, linear thermo-poroelastic fields in a saturated porous medium under radial and spherical symmetry. More recently, Osman and Rouainia [8] proposed an analytical solution for consolidation around spherical cavity contraction with the initial excess pore pressure immediately after the contraction of the cavity evaluated from the cavity contraction theory using a linear-elastic- perfectly-plastic soil model.

A review of the previous work shows that the problem of consolidation around an expanded spherical cavity and in more sophisticated soil such as Modified Cam Clay (MCC) model has not been considered. Although Osman and Rouainia [8] investigated a very similar problem of consolidation around contracting spherical cavity, his solution is derived based on the assumption of cavity expansion in linear-elastic-perfectly-plastic soil model. In addition, Randolph et al.'s closed-form solution for cylindrical cavity is also derived using the linear-elastic- perfectly-plastic cavity expansion model. Therefore, a new general analytical solution for the consolidation around expanded cylindrical or spherical

* Corresponding author at: Key Laboratory of New Technology for Construction of Cities in Mountain Area, College of Civil Engineering, Chongqing University, Chongqing 400045, China.

E-mail addresses: zh4412517@163.com (H. Zhou), hliuhhu@163.com (H. Liu), zhayahui@163.com (Y. Zha), yinf1988@163.com (F. Yin).

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Nomenclature

a_0	initial radius of spherical cavity
a	radius of expanded spherical cavity
c	conventional one-dimensional consolidation coefficient
e	void ratio
G	shear modulus of soil
K	bulk modulus of soil
M	slope of the critical state line;
p'	mean effective stress
p'_c	yield pressure under isotropic compression
q_0	initial deviator stress
q	deviator stress
r_p	radial position of the elastic-plastic (EP) boundary
r_c	radius of the critical state zone
(r, θ)	polar coordinate variables
(r, θ, φ)	spherical coordinate variables
(r_0, θ)	initial position of a soil particle around the cylindrical cavity wall
(r_0, θ, φ)	initial position of a soil particle around the spherical cavity wall
R	overconsolidation ratio
u	radial displacement
u_0	initial pore pressure
u_w	pore pressure
v'	effective Poisson's ratio
w	radial velocity component of soil particle
W	velocity at the elastic-plastic boundary
σ_0	initial stress
σ_a	cavity wall pressure
σ_r, σ_θ and σ_φ	stress components in spherical coordinate system
σ_r, σ_θ	stress components in cylindrical coordinate system
ε_p	volumetric strain
ε_q	shear strain corresponding to deviator stress
ε_r, ε_θ	strain components in polar coordinate system
v'	effective Poisson's ratio
κ	slope of unloading-reloading line in υ-ln p'plane
υ	specific volume
υ_0	initial specific volume
λ	slope of normal compression line in υ-ln p' plane
Γ	specific volume at unit p' (kPa) on the critical state line in υ-ln p' plane
η	similarity variable

Notional meaning of superscripts and subscripts

$'$	effective
p	at the elastic-plastic boundary
m_v	volume compressibility and is equal to $2G(1-v)/(1-2v)$
k_w	permeability of the soil
γ_w	unit weight of water

cavity in a more advanced soil model (MCC model) is necessary. Such an analytical solution provides a theoretical framework for predicting the dissipation of excess pore pressure around the expanded cavity, and also provides a valuable benchmark for verifying the FEM program. In addition, the solution can be used to interpret the problem of pile foundation, in-situ test, tunnel construction, compaction grouting, and so forth.

2. Basic assumptions and definition of the problem

Fig. 1 shows an initial spherical (or cylindrical, it is not presented in the figure) cavity with initial radius a_0 and an initial uniform inner pressure σ_0 expands to another cavity with radius equal to a as inner pressure increases from σ_0 to σ_a. The soil response is elastic when the pressure is relatively small. Further increase in the cavity wall pressure will result in the formation of critical state and plastic zones around the cavity wall. The soil beyond the plastic zone is in elastic state and it is defined as elastic zone. r_p denotes the radial position of the elastic-plastic (EP) boundary, which is occupied by the soil particle initially at r_{p0}. The radius of the critical state zone is r_c. The soil is assumed as homogeneous. The plastic behavior of the soil is described by the MCC model, while the elastic behavior is governed by the Hooke's law. The condition of cylindrical and spherical symmetry holds in the cylindrical and spherical cavity expansion respectively and thus cylindrical and spherical polar coordinate systems are used for cylindrical and spherical cavity expansion respectively. The initial position of a soil particle around the cavity wall can be defined as (r_0, θ) for cylindrical cavity and (r_0, θ, φ) for spherical cavity. Due to the symmetry, the stresses variable σ_θ and σ_φ for spherical cavity problem are the same, which is true for the strain and displacement variables. Under this condition, only the physical variable (stress, strain and displacement) in r and θ directions are necessary to be considered for spherical cavity, which is similar to cylindrical cavity problem. In addition, two main assumptions are used in the following derivation. The first assumption is that the distribution of the excess pore water pressure immediately after the cavity expansion can be calculated through the solution for undrained cavity expansion in MCC model. In addition, since the soil is moving back towards the cavity wall (having been originally displaced outwards during cavity expansion process) most of soil will go through a process of unloading in shear. Therefore, it is reasonable to assume that during the consolidation process the soil skeleton deforms elastically and is governed by Darcy's law. This assumption was validated by Randolph and Wroth [9] and Carter [1].

3. Solution for undrained cylindrical or spherical cavity expansion in infinite MCC model

3.1. Definition of basic variables

Following Collins and Stimpson [5], Collins and Yu [6], it is convenient to define the two stress variables p' and q with respect to the radial and tangential effective stress (σ'_r and σ'_θ) according to:

$$p' = \frac{\sigma'_r + k\sigma'_\theta}{1+k} \tag{1}$$

$$q' = \sigma'_r - \sigma'_\theta \tag{2}$$

where $k = 1$ for cylindrical cavity and $k = 2$ for spherical cavity.

The corresponding strains ε_p and ε_q are written as:

$$\varepsilon_p = \varepsilon_r + k\varepsilon_\theta \tag{3}$$

$$\varepsilon_q = \frac{k}{1+k}(\varepsilon_r - \varepsilon_\theta) \tag{4}$$

Based on the strain-displacement relation, the radial strain (ε_r) and tangential strain (ε_θ) are expressed as:

$$\varepsilon_r = -\frac{\partial u}{\partial r} \tag{5}$$

$$\varepsilon_\theta = -\frac{u}{r} \tag{6}$$

where u is the radial displacement and r is the radial position.