Computers and Geotechnics 89 (2017) 1–8

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Analytical study of fluid flow modeling by diffusivity equation including the quadratic pressure gradient term

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ARTICLE INFO

Article history:
Received 5 November 2016
Received in revised form 5 March 2017
Accepted 1 April 2017

Keywords:
Diffusivity equation
Well testing
Fluid flow
Porous media
Quadratic term
Nonlinear

ABSTRACT

Diffusivity equation which can provide us with the pressure distribution, is a Partial Differential Equation (PDE) describing fluid flow in porous media. The quadratic pressure gradient term in the diffusivity equation is nearly neglected in hydrology and petroleum engineering problems such as well test analysis. When a compressible liquid is injected into a well at high pressure gradient or when the reservoir possess a small permeability value, the effect of ignoring this term increases. In such cases, neglecting this parameter can result in high errors. Previous models basically focused on numerical and semi-analytical methods for semi-infinite domain. To the best of our knowledge, no analytical solution has yet been developed to consider the quadratic terms in diffusivity PDE of one-dimensional unsteady state fluid flow in rectangular coordinates and finite length.

Due to the resulting errors, the nonlinear quadratic term should also be considered in the governing equations of fluid flow in porous media. In this study, the Fourier transform is used to model the one-dimensional fluid flow through porous media by considering the quadratic terms. Based on this assumption, a new analytical solution is presented for the nonlinear diffusivity equation.

Moreover, the results of linear and nonlinear diffusivity equations are compared considering the quadratic term. Finally, a sensitivity analysis is conducted on the affecting parameters to ensure the validity of the proposed new solution. The results demonstrate that this nonlinear PDE is also applicable for hydraulic fractured wells, and well test analysis of fractured reservoirs.

1. Introduction

Pore pressure distribution in a single phase porous media is highly important for researchers in the fields of hydrology and petroleum reservoir engineering and also applicable for well testing analysis. This pressure distribution which has received particular interest in geophysical science, is obtained based on the transient fluid flow through porous media by diffusivity equation in a nonlinear form [1–3].

Due to the compressibility of fluids, a quadratic pressure gradient term appears in the governing PDE, which leads to high nonlinearity in diffusivity [4].

Many researchers have solved this equation by neglecting the nonlinear part of the equation to linearize it for various boundary conditions. This approach has been widely used in pressure transient analysis of porous media for small pressure gradient conditions; however, in many actual field applications, the working pressure of injection fluid into porous media is high enough that linear form of diffusivity equation deviates from the reality. Such application can be observed in many reservoir operations such as hydraulic fracturing, large drawdown flows, slug testing, drill-stem testing, and large pressure pulse testing [5,6]. In these cases, the nonlinear form of pressure distribution equation should be applied through the quadratic pressure gradient term to eliminate the errors rising from the linearization. Even though the importance of this nonlinear pressure term has been noticed in well test applications, modern well test analysis does not consider its impact on their interpretation [7].

Odeh and Babu [8] used the analytical solution of nonlinear diffusivity equation in three categories including constant rate inner boundary, infinite outer boundary and no wellbore storage effects. Their findings indicate that for the most of the reservoir engineering processes, linear form of the PDE can be used directly with small errors without quadratic pressure gradient term. However, in the case of high pressure injection, linear form of PDE causes high deviation from the reality and thus the quadratic term should be considered in the equation to solve it more accurately.

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http://dx.doi.org/10.1016/j.compgeo.2017.04.001

Nomenclature

A_c	cross section area [m^2]
erfc	complementary error function
c_f	formation compressibility [psi^{-1}]
c_t	total compressibility [psi^{-1}]
c_n	series coefficient
h	reservoir thickness [m]
K	rock absolute permeability [md]
L	reservoir length [m]
n	rock porosity
P	pressure [psi]
q	wellbore flow rate [m^3/day]
t	time [s]
w	variable change term
x_f	fracture half-length [m]
x	horizontal coordinate
$\int_{x_D=0}^{x_D=1} \sin(\lambda_m x_D)\, dx_D$	integral factor

Greek

μ	fluid viscosity [cp]
λ	eigenvalue

Subscripts

D	dimensionless
i	initial
ss	steady state
w	well
uss	unsteady state

Finjord and Aadnoy [9] presented steady state and pseudo steady state solutions of nonlinear pressure distribution equations to determine the wellbore pressure. Logarithmic transformation was introduced to present wellbore pressure solutions to the nonlinear problem. Several other authors followed similar approaches in their studies [6,8,10].

Later on, Wang and Dusseault [11] worked on this solution to provide boundary conditions by linearizing the PDE in terms of density. This approach was similar to those presented by Muskat [12].

Renshi et al. [13] derived the quadratic pressure gradient equation for dual porosity reservoirs. They concluded that this nonlinear term affects the transient well test analysis of naturally fractured reservoirs. This influence draws more attention in the type curve analysis of fluid flow behavior in low permeability, thin bed and heavy oil reservoirs.

Guo et al. [14] introduced a semi-analytical approach to solve nonlinear flow model for liquids and utilized a numerical method to solve nonlinear equation of gas flow in underground reservoirs. It was shown that the nonlinearity effect in flow equation increases as either the time passes or nonlinear coefficient increases, and is more significant in low permeable heavy oil-bearing reservoir.

Wang et al. [15] developed a transient solution of nonlinear fluid flow equation for multiple zone composite reservoirs and matched the results with real well test data of a sandstone reservoir.

Liu et al. [16] expressed that existence of nonlinear term in the flow equation results in more pressure drop than the predicted value by neglecting the nonlinear term. This phenomena was also observed in the study carried out by Li [17].

In this study, an analytical solution is presented to solve the nonlinear PDE of unsteady state diffusivity equation in linear geometry of porous media during either production or injection of compressible fluids. Two types of inner boundary conditions including constant pressure and constant rate are investigated. Finite, close, and constant charge are chosen as outer boundary conditions. While previous studies solved diffusivity equation with quadratic gradient term in the infinite acting reservoirs, closed boundary reservoirs which are closer to the reality with finite drainage area, are considered to be solved in this study. In addition, they have studied nonlinear equations geometry of radial flow in reservoirs. Cartesian coordinate corresponding to the orthogonal cross sectional area for fluid flow is another geometry of fluid flow in porous media, especially in linear flow of near fracture medium. In this study, Cartesian coordinate is considered as the base to solve the nonlinear flow equation in porous media.

2. Mathematical expression of the phenomena

Fluid flow equation of compressible fluid in linear geometry of porous media including the effect of quadratic pressure gradient term is presented below. This equation has been derived in appendix which is similar to the derivation process reported by Stewart [18,19].

$$\frac{\partial^2 P}{\partial x^2} + c_f \left(\frac{\partial P}{\partial x}\right)^2 = \frac{n\mu c_t}{K}\frac{\partial P}{\partial t} \quad (1)$$

In which P is the fluid pressure through porous media, x is the horizontal coordinate, c_f is the fluid compressibility, c_t is the total system compressibility, μ is the fluid viscosity, n is the porosity, K is the absolute permeability, and t is the time. Quadratic term in Eq. (1) has been neglected in linear analyses of previous studies due to the small pressure gradient assumption and small compressibility of the liquid [4]. This term should be considered during high production rates of compressible liquid corresponding to the high pressure gradient, without which lots of errors are caused in predicting the reservoir behavior.

The linear flow regime associates to the fluid flow around the induced hydraulic fracture in the porous media. In Fig. 1, a vertical fracture extended around the vertical well in a tight gas reservoir has been shown. The fracture medium is assumed to be of infinite-conductivity which corresponds to no pressure drop along the fracture surface.

In this study, the nonlinear PDE of fluid flow for linear geometry of porous media considering the effect of quadratic term has been solved with two different boundary conditions:

2.1. Constant wellbore flowing pressure case

In order to solve this nonlinear PDE equation, the following variable changes are used to transfer the governing equation to the dimensionless form [18,21–23]:

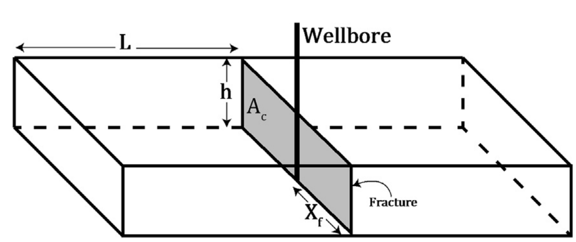

Fig. 1. Schematic of hydraulic fractured well in a linear reservoir [20].