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Semi-analytical solutions to one-dimensional consolidation for unsaturated soils with symmetric semi-permeable drainage boundary

Lei Wang [a], De'an Sun [b,*], Linzhong Li [c], Peichao Li [c], Yongfu Xu [d]

[a] College of Urban Railway Transportation, Shanghai University of Engineering Science, Shanghai 201620, China
[b] Department of Civil Engineering, Shanghai University, Shanghai 200444, China
[c] College of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai 201620, China
[d] Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

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ABSTRACT

This paper presents semi-analytical solutions to Fredlund and Hasan's one-dimensional consolidation for unsaturated soils under symmetric semi-permeable drainage boundary conditions. Two variables are introduced to transform two coupled governing equations of pore-air and pore-water pressures into an equivalent set of partial differential equations, which are easily solved by the Laplace transform. Then, the pore-air and pore-water pressures, and soil settlement are obtained in the Laplace domain. Crump's method is adopted to perform the inverse Laplace transform in order to obtain semi-analytical solutions in time domain. It is shown that the present solution is more applicable to various types of drainage boundary conditions, and in a good agreement with existing solutions from the literature. Furthermore, several numerical examples are provided to investigate the consolidation behavior of an unsaturated single-layer soil with traditional drainage boundary (single or double), and single-sided and double-sided semi-permeable drainage boundaries. Finally, it illustrates the changes in pore-air and pore-water pressures and soil settlement with time at different values of symmetric semi-permeable drainage boundary conditions parameters. In addition, parametric studies are conducted by the variations of pore-air and pore-water pressures at different ratios of air-water permeability coefficient and the depth.

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1. Introduction

The drainage boundary for one-dimensional (1D) consolidation of soil layer is generally considered as fully permeable or impermeable one. In fact, a cushion covered on the top surface of soil layer to accelerate the drainage speed should be treated as a semi-permeable drainage boundary, which is neither fully permeable boundary, nor impermeable boundary [1]. In addition, the underlying layer also has semi-permeable characteristic and can be considered as the semi-permeable drainage boundary. Therefore, the symmetric semi-permeable drainage boundary exists in many engineering practices. Furthermore, great progresses on the 1D consolidation of saturated soils under symmetric semi-permeable drainage boundary conditions have been achieved by Gray [2], Schiffman and Stein [3] and Xie et al. [1], etc. However, the study on consolidation of unsaturated soils is more general, and the solution to 1D consolidation of unsaturated soils under the symmetric semi-permeable drainage boundary conditions is rarely found in the literature. This paper aims to solve the consolidation equations for unsaturated soils under such drainage boundary.

The consolidation of unsaturated soils as a common issue in geotechnical engineering has attracted much attention of geotechnical engineering researchers, and a considerable amount of research has been conducted, and great progress has been achieved. Scott [4] estimated the consolidation of unsaturated soils with occluded air bubbles, and about which Biot [5] proposed a general consolidation theory. Barden [6] presented an analysis of 1D consolidation of compacted unsaturated clay. Furthermore, on the basis of the hypothesis that the air and water phases are continuous, Fredlund and Hasan [7] proposed a 1D consolidation theory, in which two partial differential equations are employed to describe the dissipation processes of excess pore-water and pore-air pressures in unsaturated soils. This theory is now widely accepted, and then was later extended to three-dimensional case by Dakshanamurthy et al. [8]. For simplicity, assuming all the soil parameters remain constant during consolidation, Fredlund and

* Corresponding author.
E-mail address: sundean@shu.edu.cn (D. Sun).

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Nomenclature

C_a	interactive constant with respect to air phase
C_w	interactive constant with respect to water phase
C_v^a	coefficient of volume change with respect to air phase
C_v^w	coefficient of volume change with respect to water phase
C_σ^a	consolidation coefficient for air phase
C_σ^w	consolidation coefficient for water phase
g	gravitational acceleration
h	thickness of soil layer
h_0	thickness of top boundary
h_1	thickness of bottom boundary
k_a	coefficient of air permeability
k_w	coefficient of water permeability
k_{a0}	coefficient of air permeability at top boundary
k_{w0}	coefficient of water permeability at top boundary
M	molecular mass of air
m_{1k}^a	coefficient of air volume change with respect to a change in $\sigma - u_a$
m_2^a	coefficient of air volume change with respect to a change in $u_a - u_w$
m_{1k}^w	coefficient of water volume change with respect to a change in $\sigma - u_a$
m_2^w	coefficient of water volume change with respect to a change in $u_a - u_w$
n_0	initial porosity
$Q(s)$	result of Laplace transform of $\partial q(t)/\partial t$ upon time t
q_0	initial surcharge
R	universal gas constant
R_t	parameter of semi-permeable drainage at top boundary
R_{ta}	parameter of semi-permeable drainage for air at top boundary
R_{tw}	parameter of semi-permeable drainage for water at top boundary
R_b	parameter of semi-permeable drainage at bottom boundary
R_{ba}	parameter of semi-permeable drainage for air at bottom boundary
R_{bw}	parameter of semi-permeable drainage for water at bottom boundary
S_{r0}	initial degree of saturation
T	absolute temperature
t	time
u_a	pore-air pressure
u_{atm}	atmospheric pressure
$\bar{u}_a^0$	absolute initial pore-air pressure
u_a^0	initial excess pore-air pressure
u_w	pore-water pressure
u_w^0	initial excess pore-water pressure
w	settlement
w^*	normalized settlement
z	depth
γ_w	unit weight of water
ε_v	volumetric strain

Rahardjo [9] presented a simplified form of 1D consolidation equations for unsaturated soils.

Based on the 1D consolidation theory of unsaturated soils proposed by Fredlund and Hasan [7], Qin et al. [10,11], Shan et al. [12,13], Zhou et al. [14,15], and Ho et al. [16–18] obtained several analytical solutions by different mathematical methods. Qin et al. [10,11] gave a series of semi-analytical solutions by the Laplace transform and Cayley-Hamilton technique, but the processes of derivation and simplification of the top surface state variables are very complicated. Shan et al. [12,13] also presented exact solutions of 1D consolidation for unsaturated soils under single, double and mixed drainage boundary conditions by the separation of variables method. However, the final equations have been left undisclosed as the result of cumbersome derivation, and his solutions were obtained from complex mathematical process and difficult to be used by engineers. Zhou et al. [14] got analytical solutions for single and double drainage boundary conditions by employing two alternative terms ϕ_1 and ϕ_2 to convert the nonlinear inhomogeneous PDEs into traditional homogeneous PDEs and using the separation of variables method to solve them, and the authors also obtained a numerical solution to 1D consolidation of unsaturated soils by the differential quadrature method (DQM) [15]. Ho et al. [16–18] presented a simple yet precise analytical solution of 1D, 2D and 3D consolidations of unsaturated soils under homogeneous boundary conditions by adopting eigen-function expansion method and Laplace transform technique. At present, the solution to 1D consolidation of unsaturated soils under symmetric semi-permeable drainage boundary conditions has not been obtained.

It is noted that the semi-permeable drainage boundary (i.e., so-called the third drainage boundary), which can also realize the Dirichlet and Neumann drainage boundaries (i.e., so-called the first and second drainage boundaries) by changing the values of the semi-permeable boundary parameters, are more general and practical for 1D consolidation of unsaturated soils. In addition, the existing solutions proposed in Refs. [10–18] were obtained only for the Dirichlet and Neumann drainage boundaries. Therefore, this paper presents semi-analytical solutions to the pore-air and pore-water pressures, and soil settlement of unsaturated soils deposit using the 1D consolidation theory proposed by Fredlund and Hasan [7] under symmetric semi-permeable drainage boundary conditions. To obtain final solutions, inhomogeneous governing equations for unsaturated soils are first derived into the homogeneous equations, and then the equations are solved by using the Laplace transform. It is found that the current solutions are more general and in a good agreement with the existing solutions in literature. Finally, several examples are given to illustrate consolidation behavior of unsaturated soils. Changes are sufficiently demonstrated in the pore-air and pore-water pressures and soil settlement under varying parameter values of symmetric semi-permeable drainage boundary conditions, the ratio of air to water permeability and the depth.

2. Mathematical model

2.1. Governing equations

Fredlund and Hasan [7] proposed 1D consolidation equation for unsaturated soils, in which an unsaturated soils layer is considered in infinite horizontal extent with thickness h under vertical loading q_0 and double sided semi-permeable drainage boundary conditions. The water flow, air flow and settlement only occur in the vertical direction. k_a and k_w are the coefficients of water and air permeability in an unsaturated soils layer. At top boundary, h_0 is the thickness of the sand cushion, and k_{a0} and k_{w0} are the coefficients of water and air permeability, which are obtained by the

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