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Research Paper

Effects of spatial autocorrelation structure of permeability on seepage through an embankment on a soil foundation

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ABSTRACT

Theoretical autocorrelation functions (ACFs) are generally used to characterize the spatial variation of permeability due to the limited number of site investigation data. However, many theoretical ACFs are available in the literature, and there are difficulties in selecting a suitable ACF for general cases. This paper proposes using the random finite element method to investigate the effects of ACF on the seepage through an embankment. Five commonly used ACFs—the squared exponential (SQX), single exponential (SNX), second-order Markov (SMK), cosine exponential (CSX) and binary noise (BIN) ACFs in the literature—are compared systematically by a series of parametric studies to investigate their influences on the seepage flow problem. Both stationary and non-stationary random fields are considered in this study. The results show that the commonly used SQX and SNX ACFs may overestimate and underestimate the seepage flow rate, respectively. It is also known that the maximum exit gradient associated with the SNX ACF is larger than those obtained using the other four ACFs. Additionally, it is proved that the deterministic approach-based design is on the conservative side and tends to be too conservative when dealing with soils with greater variation in the properties. It is also found that the SQX ACF has a higher probability of providing a more conservative design in practice. Overall, the differences between different ACFs are not significant and are within acceptable ranges.

1. Introduction

Accurate estimation of the seepage flow through a soil embankment is important towards the assessment of the safety of an embankment. In general, deterministic approaches that consider the soil permeability as a constant for a specific soil layer are employed to perform the seepage analysis [1]. However, due to the depositional and post-depositional processes, the soil permeability generally varies from point to point, even in a “homogeneous” soil layer. Such uncertainty (i.e., spatial variation) of the soil permeability should be explicitly incorporated into the seepage analysis model.

To date, increasing attention has been paid to the probabilistic analysis of seepage considering the spatial variation of soil permeability [1–8], since the pioneering works by Griffiths and Fenton [9] and Fenton [10]. For example, Fenton and Griffiths [4,6] investigated the effects of the spatial variation of soil permeability on the statistics of seepage through an earth dam using the random finite element method (RFEM). Before long, Griffiths and Fenton [3,5] studied the stochastic nature of the steady seepage beneath a single sheet pile wall embedded in a spatially variable soil using three-dimensional (3D) finite element analysis and the Monte Carlo simulation (MCS). Gui et al. [1] employed a probabilistic approach to explore the effects of seepage on the slope stability of an embankment, where the hydraulic conductivity was modelled as a spatially stationary random field following a lognormal distribution. Ahmed [7] proposed combining MCS with anisotropic random fields to investigate the free surface flow through earth dams. Srivastava et al. [8] quantified the influence of the spatial variability of permeability on the steady-state seepage flow and slope stability analysis. Cho [2] developed a probabilistic seepage analysis approach that accounted for the uncertainties and spatial variation of the hydraulic conductivity in a layered soil profile, and

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an earth embankment on a soil foundation was taken as an example to investigate the effects of the globally non-stationary random field of permeability on the steady seepage flow.

According to all these studies, it is found that the spatial variability of permeability has been explicitly considered in steady seepage analysis during recent years. However, these previous works still suffer from some serious deficiencies, which should be addressed. The deficiencies include the following: (1) most of the works modelled the spatial variation of permeability as an isotropic random field, where the scale of fluctuation (SOF) in the horizontal direction was the same as that in the vertical direction. Nevertheless, in reality, the soil properties fluctuate more in the vertical direction than the horizontal direction due to the natural stratification and deposition of soil deposits; (2) the random fields underlying the soil permeability are commonly assumed to be globally stationary because only a "homogeneous" soil layer was considered in most of these studies, except for the works by Cho [2]. However, multiple soil layers are commonly found in practice, where the stationary random field is no longer applicable. Instead, the hydraulic conductivities are globally non-stationary; and (3) the limitation in using the theoretical single exponential (SNX) autocorrelation function (ACF), which are commonly employed to characterize the spatial variation of permeability when site investigation data are limited. However, many other theoretical ACFs are available in the literature and may provide different simulation results that are critical to the safety property and geotechnical design of the embankment. Hence, the results obtained from different ACFs should be compared systematically.

The major objective of this study is to investigate the steady seepage through an embankment on a soil foundation using RFEM. The random permeability field was simulated using the extended Cholesky decomposition technique. Both situations of stationary and non-stationary random fields considering anisotropic heterogeneity are investigated. Five commonly used ACFs—squared exponential (SQX), SNX, second-order Markov (SMK), cosine exponential (CSX) and binary noise (BIN) ACFs—are summarized in Table 1 [11]. These ACFs were investigated systematically to explore the effects of the various ACFs on the steady seepage through an embankment. Hence, the present work will address the three limitations of the present development as mentioned in the previous paragraph.

To achieve these objectives, the rest of this paper is organized as follows. Section 2 introduces the deterministic and stochastic seepage analyses in the two-dimensional (2D) domain, followed by the discretization of the stationary and non-stationary random fields to characterize the spatial variation of permeability using the extended Cholesky decomposition technique in Section 3. In Section 4, an embankment on soil foundation is taken as an illustrative example to investigate the effects of various ACFs on the seepage through the whole system based on stationary and non-stationary random fields. The conclusions are drawn in Section 5.

Table 1
Common ACFs for characterizing the spatial variation of permeability.

ACF types	ACF expressions in 2D domain
SQX	$\rho(\tau_x, \tau_y) = \exp\left[-\pi\left(\frac{\tau_x^2}{\delta_h^2} + \frac{\tau_y^2}{\delta_v^2}\right)\right]$
SNX	$\rho(\tau_x, \tau_y) = \exp\left[-2\left(\frac{\tau_x}{\delta_h} + \frac{\tau_y}{\delta_v}\right)\right]$
SMK	$\rho(\tau_x, \tau_y) = \exp\left[-4\left(\frac{\tau_x}{\delta_h} + \frac{\tau_y}{\delta_v}\right)\right]\left(1 + \frac{4\tau_x}{\delta_h}\right)\left(1 + \frac{4\tau_y}{\delta_v}\right)$
CSX	$\rho(\tau_x, \tau_y) = \exp\left[-\left(\frac{\tau_x}{\delta_h} + \frac{\tau_y}{\delta_v}\right)\right]\cos\left(\frac{\tau_x}{\delta_h}\right)\cos\left(\frac{\tau_y}{\delta_v}\right)$
BIN	$\rho(\tau_x, \tau_y) = \begin{cases} \left(1 - \frac{\tau_x}{\delta_h}\right)\left(1 - \frac{\tau_y}{\delta_v}\right) & \text{for } \tau_x \leqslant \delta_h \text{ and } \tau_y \leqslant \delta_v \\ 0 & \text{otherwise} \end{cases}$

2. Seepage analysis

2.1. Deterministic analysis based on FEM

The steady seepage problem in the 2D domain is governed by a Laplace's equation that is derived based on the assumption that the saturated-unsaturated flow obeys Darcy's law [12]. In Cartesian coordinates, the equation is written as:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = 0 \tag{1}$$

where h is the piezometric head that is equal to the summation of the pressure head and the elevation head, and k_x and k_y are the hydraulic conductivities in the horizontal and vertical directions, respectively.

For saturated–unsaturated flow, the hydraulic conductivity depends highly on the degree of saturation or the matric suction in unsaturated soils. In general, the hydraulic conductivity function can be estimated by empirical and semi-empirical expressions in the literature. Following Van Genuchten [13], however, a set of closed-form equations as given in Eqs. (2) and (3) were used in this study to describe the soil hydraulic conductivity functions as

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + (\alpha\psi)^n\right]^m}, \quad \left(m = \frac{n-1}{n}, n > 1\right) \tag{2}$$

$$k = k_s S_e^{1/2}\left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \tag{3}$$

where S_e is the effective water saturation; θ_r is the residual volumetric water content, θ_s is the saturated volumetric water content; ψ is the matric suction or the negative pore water pressure; m, n and a are retention parameters; k is the hydraulic conductivity function; and k_s is the saturated hydraulic conductivity.

To solve Eq. (1), numerical methods, such as FEM and FDM (finite element and difference methods), are commonly adopted in the literature. In the present study, an iterative FEM was utilized to obtain the numerical results, which terminates when the difference between the results from successive iterative steps is within the predefined limit. The specific solution process of this method can be found in Fredlund and Rahardjo [14] and Cho and Lee [15]. Once the equation is solved, the seepage outputs relating to the flow rates and exit gradients can be easily obtained.

2.2. Probabilistic analysis based on MCS

It is preferable to perform probabilistic seepage analysis due to the stochastic nature of permeability. In the current study, the spatial variation of permeability was considered and incorporated into the FEM model to establish the stochastic analysis model. The spatial variation of the permeability was simulated based on a 2D random field generator, which will be illustrated later. MCS was then performed to repeat the stochastic analysis model analysis, based on which the statistics of seepage outputs, such as the means and standard deviations, are easily obtained. These results can provide more insights into the seepage through the underlying system.

3. Characterization of spatial variation of permeability

3.1. Simulation of random permeability field

Random field theory has been widely used to characterize the spatial variation of soil properties [16–18]. Within the framework of random field theory, the soil parameters at particular locations are often considered as random variables. The resultant random field is considered stationary or weakly stationary when the