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Research Paper

Multi-laminate non-coaxial modelling of anisotropic sand behavior through damage formulation

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A R T I C L E I N F O

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A modified multi-laminate model, to predict non-coaxiality in anisotropic sand, is proposed in this paper. The model can easily be extended to other geo-materials only with implementing some minor provisions. To consider anisotropy of sand, two ellipsoids are utilized to summarize shear and compressive stiffness of material in different directions. Damage concept is used to take into account degradation of material through loading procedure. Ellipsoid of rigidity factors is being changed in both size and dimension, under applied strain path. Variation of ellipsoids results in change of stiffness distribution over different planes. In other words, fabric evolution in material is considered through variation of ellipsoids of rigidity factors. A simple rule is proposed for shear stress-strain relationship in loading-unloading and reloading, which captures most of the natural characteristics of sand behavior. In multi-laminate models, depending on stiffness distribution over sampling planes, stress and strain are not coaxial essentially. To achieve better results, non-coaxiality of shear stress and strain on sampling planes is considered by applying vector field concept. Shear stress in different directions of a sampling plane is considered as a vector field. This field is obtained from strain field, considering shear stiffness in various orientations. The model parameters are calibrated using uniaxial compressive test data in different directions, with respect to bedding plane on an anisotropic sand sample. To investigate capability of the model to predict non-coaxiality, results of the model are compared to experimental results obtained from pure principal stress rotation. Ultimately, good accuracy is observed in results.

1. Introduction

Accurate prediction of deformation in geo-materials under different stress paths is crucial in solving various problems in geotechnical engineering. Principal stress rotation has a significant role on instant and permanent deformation of granular material. While principal stress magnitudes are kept constant, principal axes rotation changes fabric of the grains, and causes some displacements and deformations in the texture of the material. Gräbe and Clayton [1] showed that a realistic assessment of the total life cost of soil and ballast foundations for rail tracks requires analysis of the effects of the repeated principal stress rotation applied by trains. They explained the importance of considering principal stress rotation in evaluating permanent displacements of rail track foundations and the use of more appropriate testing methods such as the cyclic simple shear apparatus or the cyclic hollow cylinder.

During principal stress rotation, principal axes of strains rotate as well, but usually they do not coincide with each other. Predicting non-coaxiality between two sets of principal axes is one of the major concerns of researchers in recent decays. Most of the conventional models, based on tensor invariants, are not capable of considering non-coaxiality between stress and strain. These models assume that coaxiality exists between the directions of principal stresses and plastic strain increments. They are limited to loading histories in which principal stress axes do not rotate [2]. Only a part of the elasto-plastic models takes non-coaxiality into account. In geo-materials, Rudnicki and Rice [3] modified Drucker-Prager's model with non-coaxial term. Vardoulakis [4], Mehrabadi and Cowin [5] and some other researchers, modified Mohr-Coulomb's model with some changes, to consider non-coaxiality. Yatomi et al. [6] extended Cam-Clay model based on Rudnicki and Rice [3] work. Gutierrez et al. [7], Li and Dafalias [8], Lashkari and Latifi [9], Jiang et al. [10], Yang and Yu [11,12], Yu and Yuan [13] are among the recent researches who tried to include non-coaxiality in their models. For other types of models which are not based on tensor invariants, there are some attempts in literature dealing with this problem. Micro-plane models are

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among the first ones to account for stress rotation. Bažant and Oh [14] proposed a micro-plane model for brittle geo-materials that experience progressive tensile fracturing or damage. The response of the model was path dependent. The tensorial invariance restrictions were satisfied by the micro-plane model, and could be applied to progressive fracturing under rotating principal stress directions, which was the main purpose of the proposed model.

Borja et al. [15] utilized a numerical algorithm for three-invariant isotropically hardening plasticity models that took into account the rotation of principal stress axes. Anisotropy of the materials was not the matter of this study. Caner et al. [16] studied vertex effect in strain-softening concrete under nonproportional loading paths at rotating principal axes. Neher et al. [17] tried two different versions of multi-laminate models, to simulate experimental results concerning principal stress axes rotation for clay. In both models volumetric hardening is applied by using a cap surface. Qian et al. [18] presented a non-coaxial constitutive model platform in a general three-dimensional stress space, to predict shear band formation in geo-materials. Huang et al. [19] utilizing a modified Mohr–Coulomb yield function and non-coaxial non-associated flow rule, proposed a 3D non-coaxial elasto-plastic model, and validated the model by a series of true triaxial tests on loose sands. Li and Dafalias [20] described a plasticity constitutive model for inherently anisotropic sand behavior within a modified form of critical state soil mechanics. They presented fabric of the inherent anisotropic sand by a second-order symmetric fabric tensor, Fij. Thereafter, a scalar-valued anisotropic state parameter A is defined in terms of a joint invariant of fabric and stress tensor. The critical state line geometry in the plane of void ratio and effective mean normal stress is not constant, and is defined based on parameter A. This causes the other soil mechanical behavior to be a function of A as well. Similarly, Gao et al. [21] developed another failure criterion for geo-materials with cross-anisotropy. They introduced an anisotropic variable, such as previously mentioned A, into the frictional coefficient of the failure criterion, to take into account the effect of cross-anisotropy.

Influences of inherent and induced anisotropy and fabric evolution during successive load steps on non-coaxiality were not considered in all mentioned researches. In spite of few theoretical studies, carried out on principal stress axes rotation and non-coaxiality, there is large number of experimental tests in this field. Miura et al. [22] investigated deformation behavior of sand under rotational stress condition. A series of drained tests were carried out on dense anisotropic sand by means of hollow cylinder apparatus. Results showed that the shear deformation of specimens, under rotational principal stress axes condition, is large in comparison with fixed axes condition [22,23,25–27]. In addition, the influences of inherent anisotropy on the shear deformation and volumetric changes are considerably higher than the fixed axes condition. Cai et al. [23] investigated the results of various drained tests carried out on sands, using hollow cylinder apparatus. The experimental results provide clear indication of material non-coaxiality, specifically when the principal stress direction rotates. Zhou et al. [24] studied the accumulations of the excess pore water pressure and the deformation of intact soft clay, experimentally. Non-coaxial behavior of specimens, subjected to pure principal stress rotation, was specifically investigated in this work. All tests were conducted by using a hollow cylinder apparatus to investigate the effect of intermediate principal stress on the above mentioned factors. Tong et al. [25] conducted a series of tests on sands, to investigate the drained behavior of sands with inherent fabric anisotropy. An automatic hollow cylinder apparatus was utilized in this program, and samples were subjected to cyclic rotation of principal stress axes. The results exhibit obvious non-coaxiality between the directions of strain increment and stress. Zhou et al. [28] performed a research program on a series of undrained test specimens. Pure and cyclic principal stress rotation was carried out on intact and reconstituted clay in a hollow cylinder apparatus. Non-coaxiality was obviously observed during rotation of principal stress axes. The influences of inherent anisotropy and intermediate principal stress parameter were discovered to be less significant, compared to sand [28]. Yan et al. [29] also discussed the anisotropy and intermediate principal stress effects on deformation and pore pressure of reconstituted clay under rotational principal stress condition, based on experimental observations.

In most of the classical models, yield surfaces are defined in terms of stress tensor invariants. Therefore, the rotation of principal stress axes cannot be captured. In this regard, to modify this deficiency, translational or rotational yield surfaces may be utilized. This will add to the complexity of the model, and the formulation loses any physical interpretation. To this end, multi-laminate framework seems to be the best choice to deal with this issue. Models based on multi-laminate framework are capable of capturing the rotation of principle stress axes, inherent and induced anisotropy. In this paper, a simple rule is developed to consider fabric evolution of the material. Shear stress-strain relationship of the material is modified, and damage formulation is improved to take into account this modification.

2. Multi-laminate framework and review of basic relations

Multi-laminate and micro-plane framework and their behind theorem is explained in detail and well discussed in literature [30–36]. Both of the aforementioned models are the two main subgroups of micro-mechanic based models. These models utilize some planes, in different orientations, integrate material behavior in different directions and predict the material response at each point in macro scale [34]. The integration planes are called "micro-plane" or "sampling plane" in different models. In this method of analysis, a series of sampling planes, passing through a physical node, simulate the behavior of material. Stress-strain relationship is developed first over these planes instead of the point itself. To collect and reflect all of these stress variations at all orientations, numerical integration is being used to integrate the behavior of different planes as a whole. In multi-laminate framework, vectors of stress and strain in different planes take the place instead of tensors, and constitutive law of the material in micro scale is written based on mathematical relations between vectors, instead of tensor invariants.

For a granular mass, such as sand that supports the overall applied loads through contact friction, the overall mechanical response ideally may be described on the basis of micro-mechanical behavior of grains or particles' interconnections. Naturally, this requires the description of overall stress, characterization of fabric, representation of kinematics, development of local rate constitutive relations and evaluation of the overall differential constitutive relations, in terms of the local quantities. Representing overall stress tensor in terms of micro level stresses and condition, number and magnitude of contact forces has long been the aim of several researchers [37,38].

In an ideal case, the normal integration is considered to be the summing up of individual micro effects corresponding to infinite number of micro sampling planes. In this study, 2×17 sampling planes are utilized to approximate the sphere around a physical point. The choice of 2×17 planes for the solution of any three dimensional problem is a fair number, and the underlying reason is explained at the end of this section. To evaluate the variation of any scalar value at each point of the material, variations of that parameter in different planes, with given directions, can be summed up, with respect to the weight factor of that plane by the following well known formula: