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Demonstration of spatial anisotropic deformation properties for jointed rock mass by an analytical deformation tensor

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A B S T R A C T

This paper develops a joint deformation tensor (***JD***), which considers all of the joint's mechanical and geometrical parameters that affect the deformability of the rock mass. The method based on ***JD*** (***JD*** method) and an elastic deformation anisotropy index (EDAI) are deduced for estimating the spatial anisotropy deformation of a jointed rock mass. The numerical modeling and *in situ* true triaxial compressive experiments well verified the effectiveness of the EDAI and ***JD*** method for the rock mass containing one joint set, orthogonal joint sets or the rock mass containing any types of joint network with unity stiffness ratio.

1. Introduction

Theoretical demonstration and accumulated practice indicate that joints exert a dominant effect on the overall response of a rock mass because the general rock mass consists of two major components: the intact rock matrix and the complicated joint network [1–5]. At the 12th ISRM International Congress on Rock Mechanics in 2011, Barton emphasized that "jointed and anisotropic water-bearing rock masses most frequently represent the reality of engineering in rock" [6]. Structural joints clearly play the key role in the mechanical response of a rock mass; the deformational estimation of a jointed rock mass is still one of the most practical assumptions to be made for analyzing the stress and strain of the rock medium [7–9]. The general rock mass behavior may be assumed to be of equivalent isotropy in deformation, and it has been implemented in many geotechnical engineering applications [10–14]. Nevertheless, a discontinuous rock mass must be considered as an anisotropic material most of the time when the joint sets inside the rock mass are distributed along a few principal directions [15–19].

To make such an analysis successful, an analytical method that describes the joint network in space and presents the deformational anisotropy of an engineering rock mass is an important subject in rock mechanics. A general direct method for presenting the joints in a rock mass is the discontinuous method, examples of which include the distinct element method, manifold method, and discontinuous deformation analysis et al. [20–23]. Joints in the discontinuous modes are costly in terms of computer memory and speed, so it is difficult to apply those methods for direct computation of the large-scale engineering rock mass with abundant joints currently.

Thus, the continuum methods, which treat a rock mass as an equivalent continuum material with the anisotropic property, are still widely accepted. Many studies have discussed the analytic deformation solution for a jointed rock mass based on the continuous method, which treats rock masses as a continuum with equivalent material properties. For example, following the productive works of Singh [24], Amadei and Goodman [25], Fossum [26], Yoshinaka and Yamabe [27] and Wu [28] are regarding the equivalent elastic modulus, shear modulus and Poisson ration for jointed rock masses, Li et al. [29] recently developed an analytical formula for the compression transmitting coefficient and the shearing transmitting coefficient, Kim [30] suggested a mechanical stiffness tensor by driving local or global mechanical compliance tensors using coordinate transformation on the basis of Hooke's law, Tang et al. [31] presented a model to determine equivalent deformability parameters through regular and irregular fractures in a rock mass, and Wang and Huang [32] built a stress-strain relation for a rock mass containing multiple joint sets. These methods for deducing the elastic deformation matrix of rock mass always use the advantages of the generally elastic superposition principle which are not intuitive for describing the spatial anisotropy

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deformation property of the rock mass with various joint networks. Oda [33] once suggested a crack tensor for representing spatial fractures. This crack tensor, which can integrate all the mazy fractures and joints into a simple second-order tensor via the tensor product operation of the joint orientation vector, is well suited for numerical implementation using finite element methods.

In this paper, we develop Oda's original geometrical crack tensor and present a joint deformation tensor (***JD***) to demonstrate the spatial deformation properties of the jointed rock mass. The joint deformation tensor includes not only the spatial geometric parameters of the joints (i.e., size, persistence ratio and orientation) but also the mechanical parameters of the joints (i.e., normal and shear stiffness). A corresponding method based on the joint deformation tensor (***JD*** method) for calculating the equivalent elastic compliance tensor and a single index for estimating the anisotropic properties are also put forward for the jointed rock mass. The joint deformation tensor developed here realizes the expression for the deformation characteristics of the joint network in the rock mass with a succinct manner, which leads to a new approach for building a strict stress-strain constitutive model for jointed rock mass based on measurable parameters.

2. Joint deformation tensor

2.1. Expressions of the joint deformation tensor

A systemic expression for the distributed 3D joints is key for estimating the deformational property of a rock mass because the joints are originally factored for the deformation anisotropy of natural geomaterials. Therefore, Oda presented a fabric tensor for discontinuous geological materials. Oda's method integrated all the respective geometrical characteristics of cracks (i.e., density, size and orientation) into a quantitative tensor using tensor products of each crack's position vector, as shown in Eq. (1) [34]. The advantage of Oda's tensor is that a single variable combines all of the geometric properties of cracks in a logical manner. Thus, this quantitatively geometrical expression (***F***) provides us with a basis for deducing a mechanical deformation tensor for the jointed rock mass.

$$\boldsymbol{F} = \frac{\pi\rho}{4}\int_0^{\infty}\int_{\Omega} r^3\boldsymbol{n}\otimes\boldsymbol{n}\otimes\cdots\otimes\boldsymbol{n}E(\boldsymbol{n},r)d\Omega dr \tag{1}$$

where ρ is the crack density, which is defined as $\rho = m^{(V)}/V$; $m^{(V)}$ is a number of cracks whose centroids are located in a volume V; r is the equivalent diameter of the cracks; $\boldsymbol{n}$ is the positional vector of a crack; E ($\boldsymbol{n}$, r) is the joint probability density function of a crack's equivalent diameter and orientation vector; Ω is the whole solid angle (4π) equivalent to a unit sphere.

This fabric tensor in Eq. (1) accounts for the density, size and orientation of the cracks, but it cannot represent the actual deformability property by which the spatial cracks directly affect the equivalent anisotropic elastic deformation of the rock mass because Oda's crack tensor is only a structural representation. For the elastic constitutive equation of a rock mass containing an intact rock matrix and joints, an enhancement based on the fabric tensor is necessary for building an explicit expression of the spatial distribution of the joint network and integral deformation response of the joints. Thus, a deformational stiffness coefficient (JF) is introduced into Eq. (1) by utilizing Boltzmann's superposition principle for the linear-elastic stress-strain model [25,35,36]. The new mechanical tensor (***JD***) is embodied as a deformation tensor with a symmetric second-order format, as shown in Eq. (2). Because ***JD*** includes a joint's mechanical deformation parameters (i.e., normal stiffness and shear stiffness) and geometrical parameters (i.e., density, size and orientation), it characterizes the integrated deformation capability of all the joint networks inside a rock mass (see Fig. 1). The dimension of the ***JD*** is m^{-1} and it can be interpreted as the tensor representation of the weakening coefficient of a joint system's basic elastic stiffness when unit elastic deformation of a rock mass takes place, induced by the joint density, size, orientation and different deformation stiffness.

$$\boldsymbol{JD} = \frac{\pi\rho}{4}\int_{\Omega}\int_0^{\infty}\int_0^{\infty} r^2\cdot\frac{1}{JF}\cdot\boldsymbol{n}\otimes\boldsymbol{n}\cdot E(r,JF,\boldsymbol{n})drdJFd\Omega \tag{2}$$

where ρ is the volumetric density of joints; JF is a joint's deformational stiffness coefficient defined by Eq. (3); E (r, JF, $\boldsymbol{n}$) is the joint probability density function of the joint's size, stiffness coefficient and orientation vector.

$$JF = k_n/k_{n0} = k_s/k_{s0} \tag{3}$$

where k_{n0} and k_{s0} are the fiducial normal stiffness and shear stiffness, respectively, with the assumption of equal stiffness ratios.

The random characteristics of a natural joint, described with the joint probability density function in Eq. (2), can be simplified as the individual probabilistic density function as Eq. (4) if we assume that the parameters of joints show little related relation in Correlation Test [37–40].

$$\boldsymbol{JD} = \frac{\pi\rho}{4}\int_{\Omega}\int_0^{\infty}\int_0^{\infty} r^2\cdot\frac{1}{JF}\cdot\boldsymbol{n}\otimes\boldsymbol{n}\cdot E(r)\cdot E(JF)\cdot E(\boldsymbol{n})drdJFd\Omega \tag{4}$$

where $E(r)$, $E(JF)$ and $E(\boldsymbol{n})$ are the probability density functions of the equivalent diameter, stiffness coefficient and orientation vector, respectively.

In the research area, if every joint information is available through *in situ* measurement, the parameters of each joint can be used to replace their expression with probability density function in Eqs. (2) and (4). As a result, an accumulation format of the joint deformation tensor can obtained via discretization, as shown in Eq. (5).

$$\boldsymbol{JD} = \sum_{k=1}^{m^{(V)}}\frac{1}{V}\cdot s^{(k)}\cdot\frac{1}{JF^{(k)}}\boldsymbol{n}^{(k)}\otimes\boldsymbol{n}^{(k)} \tag{5}$$

where k is the serial number of joints; $JF^{(k)}$ and $n^{(k)}$ are the stiffness coefficient and orientation vector of the kth joint, respectively; and $s^{(k)}$ is the area of the joint, which is equal to $\pi(r^{(k)})^2/4$.

All the geological discontinuities originated from crustal historical formation activities along special compression and tension directions; thus, the rock mass's regnant joint sets, the planes of which are approximately parallel to each other in every set, show a general regular pattern [15,17,41]. Reorganization of the joints through geologic investigation and stereographic projection to obtain the regnant joint sets is an effective way of grasping the main spatial characteristics of a joint network [42–44]. For a rock mass with regular joint sets, the geometrical parameters of the joints include the joint spacing (S_m^g), joint persistence ratio ($p^{(g)}$), orientation vector ($\mathbf{n}^{(g)}$) in each joint set. For the non-persistent joint sets, the areal persistence ratio is calculated with Eq. (6).

$$p = \frac{S_1+S_2}{S_1+S_2+S_3} = \frac{S_1+S_2}{[\pi(r^{(g)})^2]/4} \tag{6}$$

where S_1 and S_2 are the area of joints, S_3 is the area of rock bridge, $r^{(g)}$ is the equivalent diameter of the persistent plane where the coplanar joints located, as shown in Fig. 2.