Computers and Geotechnics 88 (2017) 267–280

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

# Efficient subset simulation for evaluating the modes of improbable slope failure

A.P. van den Eijnden *, M.A. Hicks

*Section of Geo-Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN/P.O.Box 5048, 2600 GA Delft, The Netherlands*

ARTICLE INFO

*Article history:*
Received 25 October 2016
Received in revised form 9 March 2017
Accepted 17 March 2017

*Keywords:*
Mode of failure
Random finite element method
Slope reliability
Subset simulation

ABSTRACT

For analyzing low probability slope failures, a modified version of subset simulation, based on performance-based subset selection rather than the usual probability-based subset selection, is combined with the random finite element method. The application to an idealized slope is used to study the efficiency and consistency of the proposed method compared to classical Monte Carlo simulations and the shear strength reduction (SSR) method. Results demonstrate that failure events taking place without strength reduction have different modes of failure than stable slopes brought to failure by SSR. The correlation between sliding volume and factor of safety is also demonstrated.



## 1. Introduction

Taking account of uncertainty in slope stability analysis typically leads to a probability distribution of failure as a function of the factor of safety. Uncertainty can reside in different aspects of the analysis (e.g., problem geometry, material parameters, and so on). A stochastic description of the parameters allows the uncertainties to be accounted for in the numerical model of the slope. The stochastic characterization of the parameters through probability density functions introduces a multidimensional sampling space filled with all possible combinations of parameter values sampled from their marginal distributions. This sampling space is ideally spanned by a set of orthogonal vectors representing the independent parameters of the stochastic characterization. The probability of failure in slope stability analysis then comes down to integrating the probability density function over the failure domain of the sampling space.

As the deterministic function of the slope stability analysis is generally non-analytical, an exact integration procedure does not exist and a numerical approximation is required. Several numerical integration techniques have been proposed for this purpose, including deterministic methods such as the point estimate method [30,6], and the first and second order reliability methods (FORM and SORM respectively) [5]. These methods usually rely on a certain level of 'regularity' of the deterministic function and require the number of (independent) variables to be low.

When spatial variability is involved as part of the uncertainty, the number of independent variables for fully characterizing the distribution of material properties over the (discretized) domain of the model increases dramatically. The spatial variability is accounted for by means of random fields of material properties or state. In general, the number of variables related to spatial variability is the number of spatially varying quantities multiplied by the number of cells used for the discretization of the random field. With the increasing number of independent variables involved in the model, many numerical integration schemes can no longer integrate over the full sampling space due to the excessive computational load it would require. Moreover, the non-linearity of the deterministic function generally prevents the application of deterministic numerical methods, and statistical integration methods, including the different versions of Monte Carlo simulation (MCS), form the remaining option for approximating the integral over the sampling space.

The application of MCS to slope stability problems is rather straightforward and often cast into the framework of the random finite element method (RFEM) [12]. This method has been successfully applied in slope stability analysis accounting for spatial variability in 2D [16] and 3D [17]. In all these applications, MCS is used to sample from the entire sampling space following the marginal distribution in each dimension. This method of crude MCS becomes inefficient when the focus is on a small probability event which occupies only a small part of the sampling space, whereas

* Corresponding author.
*E-mail addresses:* a.p.vandeneijnden@tudelft.nl (A.P. van den Eijnden), m.a.hicks@tudelft.nl (M.A. Hicks).

http://dx.doi.org/10.1016/j.compgeo.2017.03.010
0266-352X/

**Nomenclature**

| | |
|---|---|
| $CoV$ | coefficient of variation |
| $E$ | Young's modulus |
| $F$ | failure event |
| $FOS_i$ | factor of safety for realization $i$ |
| $K^{(i)}$ | max. number of Markov steps per chain in subset $(i)$ |
| $K_{cl}$ | number of clusters in KMCM |
| $N$ | number of realizations |
| $N_c$ | conditional number of realizations, number of Markov chains |
| $N_t$ | total number of realizations |
| $R$ | residual term |
| $V$ | domain volume |
| $X(\vec{x})$ | random field |
| $Z$ | normalized distance from mean value (standard score) |
| $Z(\vec{x})$ | standard normal random field |
| $\Omega$ | domain |
| $\bar{\gamma}$ | variance reduction factor |
| $\gamma$ | Markov chain correlation factor |
| $\lambda^i$ | $i$th eigenvalue |
| $\mu$ | mean |
| $\mu_{LN}$ | mean of logarithm |
| $\nu$ | Poisson's ratio |
| $\sigma$ | standard deviation |
| $\sigma_{LN}$ | standard deviation of logarithm |
| $\tau$ | distance normalized against $\vec{\theta}$ |
| $\vec{\theta}$ | scales of fluctuation |
| $\vec{c}^k$ | average nodal displacement vector of cluster $k$ |
| $\vec{u}$ | (nodal) displacement vector |
| $\vec{x}$ | spatial coordinate vector |
| $c_u$ | cohesion (undrained shear strength) |
| $f_s^{end}$ | final/target strength reduction factor |
| $f_s$ | strength reduction factor |
| $m$ | total number of subdivision levels |
| $n$ | number of cells, sample space dimension |
| $p_0$ | (target) conditional probability |
| $p_f$ | probability of failure |
| $z(\vec{x})$ | random field value at position $\vec{x}$ |
| $\Psi(.)$ | (lognormal) distribution transformation function |
| $\rho(.)$ | correlation function |
| $\rho(\Omega_A, \Omega_B)$ | correlation between domains $A$ and $B$ |
| $q(.)$ | prior distribution (standard normal) |
| $\boldsymbol{I}(f_s)$ | binary failure indicator column vector |
| $\boldsymbol{X}$ | discretized random field column vector |
| $\boldsymbol{Z}$ | standard normal random field column vector |
| $\boldsymbol{\phi}^i$ | eigenvector $i$ |
| $\boldsymbol{\theta}$ | standard normal realization sample |
| $\xi$ | standard normal random number column vector |
| $\xi^{01}$ | uniform random number column vector |
| $\xi^{Ms}$ | random Markov step column vector |
| E[.] | expectation operator |
| P(.) | probability operator |
| $\mathbf{A}$ | $n \times n$ matrix |
| $\mathbf{\Lambda}$ | diagonal matrix of eigenvalues |
| $\mathbf{\Phi}$ | matrix of eigenvectors |
| $\mathbf{C}$ | covariance matrix |
| $\mathbf{L}$ | lower triangle Cholesky factor |
| $[.]^T$ | transpose |
| $[.]^p$ | proposal |
| $[.]^r$ | reduced |
| $[.]^s$ | seed |
| $[.]^{(i)}$ | subset level $i$ |
| $[.]^{MCS}$ | Monte Carlo simulation |
| $[.]^{SS}$ | subset simulation |
| $[.]^{end}$ | final/target |
| $[.]_L$ | lower bound |
| $[.]_U$ | upper bound |
| $[.]_i$ | index of dimension or realization number |
| $[.]_n$ | normalized against number of FEM calls |
| $[\bar{.}]$ | spatial average |
| $[\hat{.}]$ | estimation |
| CMD | covariance matrix decomposition |
| DSA | direct stability analysis |
| EOLE | expansion optimal linear estimation |
| FEM | finite element method |
| KMCM | $K$-means clustering method |
| LAS | local average subdivision |
| LEM | limit equilibrium method |
| MCMC | Markov chain Monte Carlo |
| MCS | Monte Carlo simulation |
| MMA | modified Metropolis-Hastings algorithm |
| PDF | probability density function |
| RFEM | random finite element method |
| SS | subset simulation |
| SSR | shear strength reduction |

more advanced sampling strategies may lead to a higher sampling density close to the domain of interest.

Recently, Li et al. [25] applied subset simulation (SS) to the modelling of small probability failure events of slopes in spatially varying soil, in which the strength reduction method was used to determine the factor of safety of each realization in the simulation. This approach determines the required ranking of the conditional simulations in order to select the next conditional subset. The need for applying the strength reduction method in each realization introduces a computational load significantly larger than when analyzing the stability of a slope for a given level of strength reduction. A possible alternative that bypasses the strength reduction method in SS can significantly improve the efficiency of the total analysis.

This paper aims at slope stability analysis for factors of safety corresponding to low probability of failure. The factor of safety *FOS* is here based on the shear strength reduction method and defined as the factor by which the shear strength of the material needs to be reduced to trigger failure. Note that *FOS* is a realization-specific property of the slope, whereas the strength reduction factor $f_s$ is a simulation parameter against which *FOS* is compared.

## 2. Slope stability analysis using the random finite element method

The random finite element method (RFEM) [12] is used to evaluate the reliability of a slope with spatially variable strength. The framework of RFEM can be subdivided into three parts:

- generating a series of realizations, according to the stochastic characterization of the problem under consideration;
- evaluating the response of the deterministic function for each realization;
- translating the resulting factors of safety *FOS* to the reliability of the slope.