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Research Paper

# Numerical simulation of kinetic friction in the fracture process of rocks in the framework of General Particle Dynamics

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ABSTRACT

A numerical simulation of kinetic friction function in the fracture process of rocks in the framework of General Particle Dynamics (GPD) is performed in this paper. The frictional algorithm is implemented into the General Particle Dynamics code (GPD) to describe frictional behavior of particles, where frictional forces among discrete particles are formulated using the principle of balance of two forces based on ideal plastic contact between two surfaces of solids. In General Particle Dynamics code (GPD), interaction among discrete particles is formulated using the virtual-bond method. Fractures of virtual bonds among particles are determined through the Hoek-Brown damage evolution law of rock materials. Three numerical cases are to verify the stability and accuracy of the numerical algorithm. Then, the numerical results are compared with analytical solutions and experimental results. It is found that the numerical results are in good agreement with the experimental ones.



## 1. Introduction

Two solid surfaces, which are in contact, will slide slowly against each other when considering the frictional effect. An intermittent vibration phenomenon can be observed in the variation of the frictional force and the relative sliding when the kinetic frictional function is considered. Better understanding of the kinetic frictional responses of rock-like materials promises benefits in many areas from rock mechanics to slope and underground engineering and earthquake prediction. When the kinetic frictional function is considered, it is important to understand how cracks nucleate and propagate and coalesce in order to provide better understanding of the fracture process of rock-like materials that occurs in the rock engineering fields. Numerical simulations proven necessary in the studies of kinetic frictional function between solid surfaces because of the mechanical and geometrical complexity of most of the discontinuous problems.

Some meshless numerical methods [1–4] use discrete particles to construct the continuous function to avoid the difficulties associated with mesh distortion in gridding arithmetic. The meshless methods always cost a lot more computing time and the accuracy of some methods is still dependent on the nodes or particles distribution to some extent. Therefore, only a few of them can perform in failure problems, such as the smoothed particle hydrodynamics (SPH) [5–7], material point method (MPM) [8,9], hybrid particle-element method [10–12], conversion of distorted elements into particles method [13–15]. Particle methods are well suited for modelling extremely large deformation problems with fractures with the help of their discretization equations, so they are superior to the Lagrangian gridbased methods in the simulation of fracture problems.

The Extended Finite Element Method (XFEM) [16], the Generalized Finite Element Method (GFEM) [17,18] and the Particle Finite Element Method (PFEM) [19,20] were developed from the Finite Element Method (FEM). The PFEM is a meshless method as GPD (General Particle Dynamics) developed by authors [21–23]. The above three methods are commonly used in the engineering problems. However, it is quite challenging for these methods to consider slipping and separation along the flaw direction when the frictional algorithm is considered. Although the previous GPD method simulated the fracture process of rock-like materials under uniaxial and biaxial compression [22,23] and the progressive destruction of rock slope [21], it did not consider the frictional function between the GPD particles.

The numerical manifold method (NMM) [24] is often used to analyze crack propagation problems. NMM is a combination of the discontinuous deformation analysis (DDA) [25] and the FEM. It is used to solve stationary crack and discontinuous problems of

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crack propagation [26]. However, the accuracy of the method is reduced when crack tips happen to stop inside the element. In order to improve the accuracy, the stress intensity factors (SIFs) can be used to accurately evaluate with a regular and relatively coarse mathematical cover system [27].

Smoothed particle hydrodynamics (SPH) is one of the earliest purely Lagrangian mesh-free methods. SPH was originally proposed by Lucy [28] and Gingold and Monaghan [29] to solve astrophysical problems. Since SPH has been invented, it has been extended and applied to a vast range of applications. SPH was developed to solve dynamic response of material strength problems [30–32], not merely fluid flows [33–35], but also heat transfer problems [36]. In SPH, an interaction between any two particles is only controlled by the kernel function and the interaction is automatically terminated if one leaves the influence domain of the other. This inherent fracture mode [37,38] cannot simulate initiation, propagation and coalescence of the flaws very well, and cannot to consider the frictional function between the surfaces of flaw. Chakraborty and Shaw [39] developed an efficient approach to model discrete crack and crack interaction in the framework of SPH considering the tensile strain in pseudo-springs. However, the shear stress and the frictional function were not taken into account in the pseudo-springs and frictional problems cannot be simulated.

In most geotechnical engineering problems, the boundary conditions are very complex, such as seepage boundary and temperature boundary. Monaghan has summarized situations that the boundaries of rigid bodies are modeled by using (i) ghost particles, (ii) fluid particles, (iii) normalizing conditions, or (iv) boundary particle forces [40]. More information can be found from works in [31,33,34,41–43]. Besides the aforementioned methods, particle-to-surface contact or particle-to-particle contact based on the momentum equation have also been used to describe contact behaviors, where contact force is applied between particle centers or along average normal of the boundary [44–46]. To our knowledges, all these above methods are applicable in perfectly smooth or non-sliding boundaries commonly occurring between rigid bodies.

From the previous literatures, the only algorithm treating the static frictional contact condition within the framework of SPH was proposed by Gutfraind and Savage [47]. The Coulomb-friction boundary was applied by means of a wall that exerts a normal potential force of repulsion on the SPH particles. The wall combined with a tangential force that is proportional to the normal force. However, it cannot be used in deformable structures. In order to resolve the static frictional contact problems in deformable structures, Wang and Chan [48] refined the algorithm to extend the application of SPH in deformable structures. However, the refined algorithm by Wang and Chan [48] can only treat the static frictional contact condition between the boundaries and deformable structures. Moreover, in the essence of Wang's model, the static friction and fracture functions in deformable structures are not considered in the refined algorithm proposed by Wang and Chan [48].

The object of this study is to propose an efficient and robust numerical method called General Particle Dynamics (GPD) [21–23], which adopts the virtual-bonds method to overcome the shortcomings of Smooth Particle Hydrodynamics, to simulate the initiation and growth of cracks with frictional contact between rock-like materials. In the simulation process, a given particle is assumed to interact only with its neighboring particles in the influence domain. It is reasonable that the kernel usually distributes a large enough weight around the particle of interest, and these particles in the domain remain as the major influence domain of interaction throughout the simulation time. A consistency preserving correction is applied when computing spatial gradient of the kernel so that the reduced support does not influence the basic functional approximation. The connectivity among these neighbors is established by virtual bonds. In the simulation process, the friction between any two particles is determined based on the damage of the bonds, and the friction is only considered when the bond is damaged. Furthermore, the bond is damaged when its stresses satisfy the Hoek-Brown strength criterion. The sequence of failure of neighboring bonds is captured to trace the propagation of cracks. Ability of the proposed model to accurately predict initiation and propagation and coalescence of cracks with frictional contact is demonstrated by a few examples.

This paper is organized as follows: In Section 2, the main steps involved in GPD are briefly outlined. In Section 3, the frictional interaction algorithm is illustrated. The damage model of GPD is derived and its implementation issues are discussed in Section 4. The geometries of the numerical model are demonstrated by some examples in Section 5. Conclusions are then drawn in Section 6.

## 2. Brief introduction to General Particle Dynamics (GPD)

The governing equation for continuum mechanics are shown as [30,31,39,49]

$$\frac{d\rho}{dt} = -\rho\frac{\partial v^{\beta}}{\partial x^{\beta}} \tag{1}$$

$$\frac{dv^{\alpha}}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} \tag{2}$$

$$v^{\alpha} = \frac{dx^{\alpha}}{dt} \tag{3}$$

where $\rho$ denotes its mass density. $x^{\alpha}$, $v^{\alpha}$ and $\sigma^{\alpha\beta}$ are respectively elements of the spatial coordinate (X), velocity vector (V) and Cauchy stress tensor ($\sigma$) with tension taken as positive one, $d/dt$ is the time derivative taken in the moving Lagrangian frame, and the superscripts $\alpha, \beta = 1, 2$ are integer indices for the two spatial directions.

## 3. The frictional interaction algorithm

### 3.1. Correction of governing equation on the surfaces of flaws

For the $i$th particle, if the body force and the contact force (i.e., the frictional force in this study) are taken into consideration, Eq. (2) is changed to be

$$\frac{dv^{\alpha}}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} + \frac{F^{\alpha}}{m} + b^{\alpha} \tag{4}$$

where $F^{\alpha}$ is the contact force along the coordinate direction of $\alpha$, $b$ is the acceleration caused by the body force, $m$ is the mass of particle, $\rho$ denotes the real time mass density of particle.

As shown in Fig. 1, the support domain of the $i$-th particle is composed of two entirely different types of particles, which located on either side of the pre-existing flaw. Two parts of the support domain are incomplete, that is, only contain $\Omega_{inner}$ and $\Omega_{outer}$, respectively. Stresses of particles in $\Omega_{outer}$ are all set to be zero. In doing so, the $i$-th particle only has an incomplete domain, which is $\Omega_{inner}$. Therefore, Eq. (4), which is derived from Eq. (2), cannot be discretized ($\partial\sigma^{\alpha\beta}/\partial x^{\beta}$). Wang and Chan [48] solved this problem. When the contact function is not considered, the equation of motion can be rewritten as

$$\frac{dv_i^{\alpha}}{dt} = \sum_{j\in\Omega_{inner}} m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} + \Pi_{ij}\right)W_{ij,\beta} + b^{\alpha} \tag{5}$$