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Research Paper

Semi-analytical solution for one-dimensional consolidation of fractional derivative viscoelastic saturated soils

Lei Wang [a], De'an Sun [a,*], Peichao Li [b], Yi Xie [b]

[a] Department of Civil Engineering, Shanghai University, Shanghai 200444, China
[b] College of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai 201620, China

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ABSTRACT

This paper presents a semi-analytical solution to one-dimensional consolidation equation of fractional derivative Kelvin-Voigt viscoelastic saturated soils subjected to different time-dependent loadings. The theory of fractional calculus is first introduced to Kelvin-Voigt constitutive model to describe consolidation behavior of viscoelastic saturated soils. By applying Laplace transform upon the one-dimensional consolidation equation of saturated soils, the analytical solutions of effective stress and settlement in the Laplace transform domain are obtained. The present solutions are more general and have good agreements with available solutions from the literature, and are degenerated into ones for one-dimensional consolidation of elastic and viscoelastic saturated soils.

1. Introduction

Based on the hypothesis of linear elasticity, Terzaghi presented one-dimensional (1D) consolidation theory of saturated soils [1]. However, Buisman first discovered that the long term deformation of soils after the excessive pore pressure was fully dissipated in experiments [2], which was also reported in the works of Zeevaert [3], Leonards and Ramiah [4], and in lots of engineering practices. This long term deformation, which cannot be explained by classical theories, is usually called creep. Since the 1960s, a number of viscoelastic constitutive models were proposed to describe the creep of clays, such as Maxwell, Kelvin-Voigt and Merchant models [5]. The viscoelastic constitutive model was introduced to the consolidation theory for saturated soils by Tan [6] in the 1950s, and some progress were made in subsequent years by Gibson and Lo [7], Lo [8], and Xie and Liu [9].

For the fractional derivative viscoelastic model, Gemant [10] first proposed a fractional derivative constitutive model for viscoelastic materials, and then the fractional derivative viscoelastic model is gradually becoming a hot research topic. Bagley and Torvik [11–13] and Koeller [14] developed Maxwell, Kelvin-Voigt, and standard linear solid viscoelastic models using the fractional calculus, respectively. Yin et al. [15] proposed a fractional variable-order creep model which could exactly correspond to the motions of pore water and the solid skeleton. Also, Yin et al. [16] derived an analytical formula under different conditions, including creep, stress-relaxation, loading and unloading, and validated the proposed model by laboratory experiments. Yin et al. [17] obtained the fractional order α of the Kelvin-Voigt viscoelastic model on the basis of 1D consolidation creep tests on a compacted soil.

Fractional calculus has been successfully applied to characterize the rheological property of viscoelastic materials. However, consolidation behavior of saturated soils was seldom involved in fractional order constitutive models. In order to study the consolidation behavior of fractional derivative viscoelastic saturated soils, this paper presents a semi-analytical solution to Terzaghi's 1D consolidation of viscoelastic saturated soil on the basis of the fractional derivative Kelvin-Voigt model. Moreover, several typical examples are given to assess the effects of fractional order, viscoelastic and loading parameters on consolidation behavior of saturated soils. It illustrates the changes in soil settlement with time factor at different values of the parameters. In addition, the classical viscoelastic models were mainly used in the studies on the consolidation behavior of saturated soils, the viscoelastic constitutive models incorporating with the fractional calculus have been well established for fairly wide range of viscoelastic materials, and the advantages of the adopted fractional calculus in viscoelasticity are that the constitutive relation of some viscoelastic materials can be described accurately by the fractional calculus model with a few experimental parameters, while the study on the

* Corresponding author.
E-mail address: sundean@shu.edu.cn (D. Sun).

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Nomenclature

A	linear loading rate
a	real number
B	parameter affecting the loading magnitude
C	parameter controlling the rate of exponential loading
E_s	modulus of compressibility
i	imaginary number
k	positive integer
N	integer
$Q(s)$	result of the Laplace transform of $q(t)$ upon time t
q	loading
q_0	final surcharge
q_1	initial surcharge
R^+	positive real number
T	real number
T_1	period
α	fractional order
Γ	gamma function
γ_w	unit weight of water
ε	total strain of fractional derivative Kelvin-Voigt constitutive model
$\varepsilon(z,t)$	strain
ε_e	strain of the spring element
ε_v	strain of the fractional derivative Abel dashpot
η	viscosity coefficient
λ	time factor
σ'	total effective stress of fractional derivative Kelvin-Voigt constitutive model
$\sigma'(z,t)$	effective stress
σ_e	effective stress of the spring element
σ_v	effective stress of the fractional derivative Abel dashpot
τ	integral variable
$\varphi(z)$	function relating to the depth z
$\psi(t)$	function relating to the time t
ω	angular frequency for sinusoidal loading function

consolidation of fractional derivative viscoelastic saturated soil is rarely reported in the literature.

2. Theory of fractional calculus

2.1. Basic definition of fractional derivative calculus

The Riemann-Liouville fractional calculus operator theory [18] is widely used in the rheological constitutive model. In the Lebesgue integral interval $L_1(0,t)$, the integral operator of fractional order q ($Re(q) > 0$) is defined by

$$\frac{d^{-q}f(t)}{dt^{-q}} = \int_0^t \frac{(t-\tau)^{q-1}}{\Gamma(q)} f(\tau)d\tau \quad q \in R^+,\ t > 0 \tag{1}$$

where Γ is the gamma function, q is the fractional order, τ is a real variable, and R^+ is positive real number.

Corresponding v order $(-1 < Re(v-n) \leqslant 0)$ differential operator is

$$\frac{d^v f(t)}{dt^v} = \frac{d^n}{dt^n}\left\{\int_0^t \frac{(t-\tau)^{n-v-1}}{\Gamma(n-v)} f(\tau)d\tau\right\} \quad v \in R^+,\ n \in N,\ t > 0 \tag{2}$$

where N is the integer.

2.2. Fractional derivative rheological element

From the definition of fractional derivative, the constitutive model of fractional derivative Abel dashpot [19] is written as follows:

$$\sigma(t) = \eta \frac{d^\alpha \varepsilon(t)}{dt^\alpha} \quad 0 \leqslant \alpha \leqslant 1 \tag{3}$$

where η is a viscosity coefficient (MPa d), α is a fractional order, and t is time (day).

If it is assumed that $\alpha = 1$, the fractional derivative rheological element can be regarded as the ideal Newton fluid; If it is assumed that $\alpha = 0$, the fractional derivative rheological element can be regarded as the linear elastic solid; if it is assumed that $0 < \alpha < 1$, the fractional derivative rheological element can be regarded as fractional derivative viscoelastic body; α can describe the property of fluid state [20]. In the engineering application, the value of α can be gotten by numerical simulation on the basis of the consolidation creep test results [17].

3. Mathematical modeling

3.1. Governing equation

The system consisting of a soil layer with viscoelastic assumption is shown schematically in Fig. 1. In the soil layer, $2H$, k_v, E_s, η and $q(t)$ represent the thickness, permeability coefficient, modulus of compressibility, viscosity coefficient and time-dependent loading, respectively.

Based on the assumption of Terzaghi's consolidation theory [1], the 1D consolidation equation under time-dependent loading can be expressed as follows:

$$\frac{\partial \varepsilon(z,t)}{\partial t} = \frac{k_v}{\gamma_w}\frac{\partial^2 \sigma'(z,t)}{\partial z^2} \tag{4}$$

where $\sigma'(z,t)$ is the effective stress; $\varepsilon(z,t)$ is the corresponding strain of $\sigma'(z,t)$; γ_w is the unit weight of water, that is γ_w = 9.8 kN/m^3.

Initial condition:

$$\sigma'(z,0) = 0 \tag{5}$$

The top and bottom boundaries are all considered to be permeable to water phase.

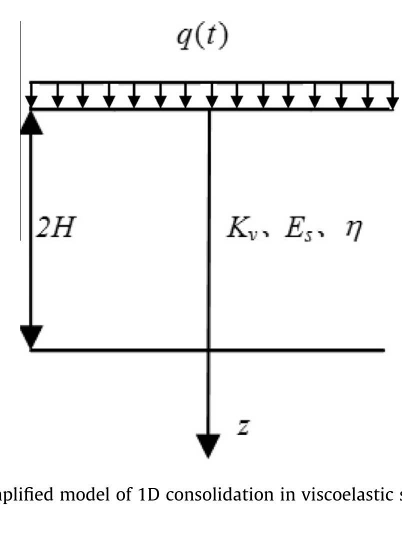

Fig. 1. A simplified model of 1D consolidation in viscoelastic saturated soils.