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Research Paper

Thermo-mechanical coupling response of a layered isotropic medium around a cylindrical heat source

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ABSTRACT

Based on the governing equations of the thermo-elastic problem, the analytical layer-elements of a finite layer and an underlying half-space are obtained using the Laplace-Hankel transform and the characteristic value method. The cylindrical heat source is divided into several micro cylindrical heat sources, which can be approximately simulated by plane heat sources. Then, the global stiffness matrix for the problem is assembled and solved in the transformed domain, and a Laplace-Hankel transform inversion is taken to obtain the real solution. Finally, the influence of heat source types, division numbers, embedded depths and layered properties on the thermo-mechanical coupling response is investigated.

1. Introduction

As one of the technologies used in energy-saving buildings, the ground heat exchanger is a key component within the geothermal heat pump system, which transfers heat to or from the ground. An energy pile is a representative ground heat exchanger [1], which has become a topical issue in geotechnical engineering. Furthermore, with the development of nuclear energy, the safe disposal of nuclear fuel waste remains a challenging task faced by the scientific and technological world [2]. In these two cases, energy piles and nuclear fuel waste repositories are typically modelled as cylindrical heat sources, and thus, studies have investigated the response of a medium around a cylindrical heat source and determined the effects of energy piles and nuclear fuel waste repositories on the surrounding media in geotechnical engineering.

Biot [3] first established the theoretical basis of thermo-elastic problems and proposed analytical solutions for isotropic and anisotropic media. Smith and Booker [4] derived the boundary integral equation of uncoupled thermo-elasticity in the Laplace transform domain and computed the thermal stresses around an infinite length heat source in a homogeneous isotropic solid. Carter and Booker [5] developed the governing equations for the fully coupled theory of thermo-elasticity and then proposed a method to solve these equations based on the finite element technique. Bai [6] studied the thermal consolidation of a cylindrical dispersing heat source with infinite length in saturated soil using the Fourier and Laplace transform methods.

In recent years, experiments have been performed to study the thermo-mechanical coupling problems due to cylindrical heat sources, such as energy piles. Moniro and Oka [7] were early researchers that used a steel pile as a heat exchanger in building foundations. Yavari et al. [8] performed a scale model experiment to study the thermo-mechanical behaviour of the heat exchanger under different thermal cycles in homogeneous isotropic soil. Huang et al. [9] investigated a model test to study the mechanical behaviour of a cylinder under cyclic temperature loads in an isotropic medium. Ng et al. [10] performed a series of centrifuge model tests in uniform saturated sand to investigate the heating effects on the settlement patterns as well as the capacities of single piles.

In actual engineering scenarios, soils around energy piles or nuclear fuel waste repositories have properties that are naturally layered. Several researchers have described the layered characteristics of media in thermo-elastic problems. Small and Booker [11] analysed the behaviour of layered soil or rock containing a decaying heat source. Pan [12] developed the propagator matrix method to analyse thermo-elastic problems in a layered medium caused by a point heat source. Bai [13] presented an analytical method for a thermal consolidation study in layered porous thermos-elastic half-space caused by a thermal loading applied on the free surface. Ai et al. [14] presented an analytical layer-element method to study the influence of point, disk and ring heat sources on layered anisotropic material. Later, Ai et al. [15] introduced an extended precise integration method for thermo-elastic problems and

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illustrated that stratification has an important effect on the thermo-mechanical response. However, the studies of thermo-elastic problems in a layered medium caused by a point heat source or a plane heat source cannot properly describe the thermo-mechanical responses of cylindrical heat sources. Therefore, many researchers have focused on studying the thermo-mechanical responses caused by cylindrical heat sources. Based on Biot's classical theory of soil consolidation [16], Selvadurai and Nguyen [17] developed a scoping analysis of coupled thermo-hydro-mechanical behaviour of a rock mass around a nuclear fuel repository and proposed a finite element formulation to compute the fractured medium consolidation around a cylindrical heat source. Laloui et al. [18] performed an in situ test of a heat exchange pile in Lausanne (Switzerland) and analysed the thermo-mechanical behaviour of the pile and the thermo-hydro-mechanical behaviour of the layered soil under thermal and mechanical loadings. Gui and Cheng [19] studied the strain and stress in a pile under thermal cyclic loading in layered soil by an in situ test in Xinyang, Henan Province (China) and established a simplified analytical model. Amatya et al. [20] synthesized the results from three published field studies and illustrated several of the engineering behaviours of such piles during heating and cooling; then, the authors discussed the effect of the end restraint and ground conditions on the thermo-mechanical response of energy piles in layered soils.

To the authors' best knowledge, few scholars have taken the layered characteristics into consideration in the numerical modelling of a medium around a cylindrical heat source. Thus, this paper aims at developing a simplified analytical method for the thermo-mechanical response of a layered isotropic medium around a cylindrical heat source based on the superposition principle. With the help of Laplace-Hankel transforms, the governing equations are solved by the characteristic value method, which is similar to the method developed by Small and Booker [11] and Booker and Savvidou [21]. Then, the cylindrical heat source is divided into several micro cylindrical heat sources, which can be approximately simulated by plane heat sources. Subsequently, the global stiffness matrix for a layered medium is assembled and solved in the transformed domain. Finally, parametric analyses are performed to study the influence of heat source types, division numbers, embedded depths and layered properties on the thermo-mechanical response of surrounding media.

2. Formulation derivation

2.1. Derivation of analytical layer-elements

A non-decaying cylindrical heat source buried in a layered thermo-elastic medium in the cylindrical coordinate system, which is infinite in the horizontal extent, is shown in Fig. 1. The heat source has a thermo-mechanical effect on the medium.

In the cylindrical coordinate system, this problem can be simplified to a axisymmetric problem about the z-axis, and the temperature increments and displacements are independent of the φ coordinate. When body forces are neglected, the equilibrium equations in a cylindrical coordinate system can be expressed as

$$\frac{\partial \sigma_r}{\partial r}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\varphi}{r}=0 \tag{1a}$$

$$\frac{\partial \tau_{rz}}{\partial r}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{rz}}{r}=0 \tag{1b}$$

where σ_r, σ_φ and σ_z denote the normal stress components in the r, φ and z directions, respectively; τ_{rz} is the shear stress in plane $r-z$.

In an isotropic thermo-elastic medium, the constitutive equations that consider temperature increments can be expressed as [22]

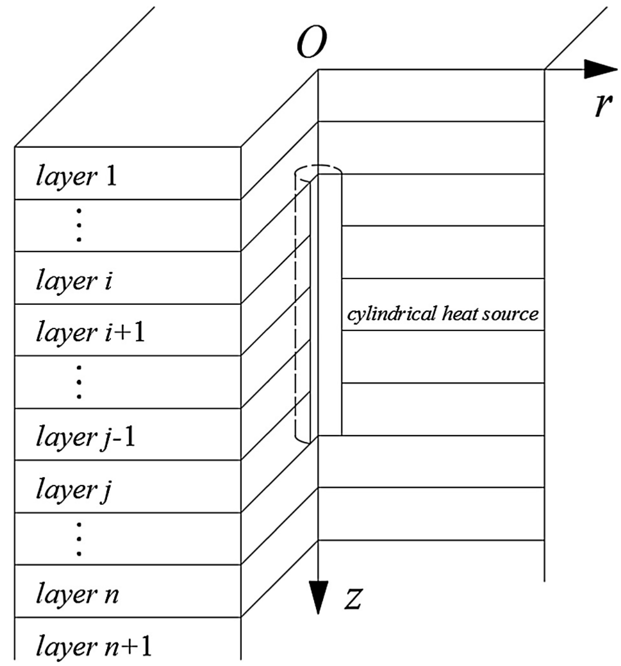

Fig. 1. Cylindrical heat source in the medium.

$$\sigma_r=\lambda\varepsilon_v+2G\frac{\partial u_r}{\partial r}-\beta\theta \tag{2a}$$

$$\sigma_\varphi=\lambda\varepsilon_v+2G\frac{u_r}{r}-\beta\theta \tag{2b}$$

$$\sigma_z=\lambda\varepsilon_v+2G\frac{\partial u_z}{\partial z}-\beta\theta \tag{2c}$$

$$\tau_{rz}=G\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right) \tag{2d}$$

where u_r and u_z represent the displacement components in the r and z directions, respectively; $\varepsilon_v=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$ represents dilatation; $\lambda=\frac{2G\mu}{1+2\mu}$ and $G=\frac{E}{2(1-\mu)}$ denote the Lamé modulus and elastic shear modulus of the medium, respectively; μ and E are the Poisson's ratio and Young's modulus of the medium; $\beta=\frac{2G\alpha(1+\mu)}{1-2\mu}$ denotes the thermal stress modulus; α and θ are the coefficient of linear thermal expansion of the medium and the temperature increment, respectively.

Substituting Eqs. (2a)–(2d) into Eqs. (1a) and (31), we obtain

$$\nabla^2 u_r+\frac{1}{1-2\mu}\frac{\partial \varepsilon_v}{\partial r}-\frac{1}{r^2}u_r-\frac{\beta}{G}\frac{\partial\theta}{\partial r}=0 \tag{3a}$$

$$\nabla^2 u_z+\frac{1}{1-2\mu}\frac{\partial \varepsilon_v}{\partial z}-\frac{\beta}{G}\frac{\partial\theta}{\partial z}=0 \tag{3b}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$ denotes the Laplacian operator.

Considering the semi-coupling of the elastic and thermal processes, the heat diffusion equation can be established as follows [5]:

$$\frac{\partial\theta}{\partial t}=a\nabla^2\theta \tag{4}$$

where $a=\frac{k}{\rho c}$ denotes the coefficient of thermal diffusivity, k is the coefficient of heat conductivity and ρ and c are the mass density and the specific heat capacity of the medium, respectively.