Construction and Building Materials 156 (2017) 761–772

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Stiffness matrix for the analysis and design of partial-interaction composite beams

Jian-Ping Lin [a], Guannan Wang [b,*], Guangjian Bao [c], Rongqiao Xu [d]

[a] College of Civil Engineering, Huaqiao University, Xiamen, Fujian Province 361021, China
[b] Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409, USA
[c] Transportation Bureau, Tonglu County, Hangzhou 311500, China
[d] Department of Civil Engineering, Zhejiang University, 866 Yuhangtang Road, Hangzhou 310058, China

HIGHLIGHTS

- Stiffness matrix is derived for partial-interaction Timoshenko composite beam.
- Explicit expressions will benefit structural calculations and designs.
- Explicitly expressed stiffness matrix has easy connection with commercial packages.
- The stiffness matrix is validated through numerical results.
- Particle swarm optimization is adopted for shear connector distribution.

ARTICLE INFO

Article history:
Received 15 March 2017
Received in revised form 18 July 2017
Accepted 29 August 2017

Keywords:
Composite beam
Partial interaction
Finite element analysis
Explicitly-expressed stiffness matrix
Interfacial slip
Shear deformation
Particle swarm optimization

ABSTRACT

Compared to the classical Rayleigh-Ritz method and other analytical solutions, finite element (FE) method is more efficient and capable in calculating the deformations and stress states of partial-interaction composite beams, as well as manipulating the material and geometrical parameters for better engineering designs. Stiffness matrix of composite beams considering the interlayer slips is derived based on the kinematic assumptions of the Timoshenko's beam theory by taking into account of the transverse shear deformations. A detailed derivation is elaborated to obtain the local stiffness matrix for a composite beam element, while the higher-order interpolation functions are adopted for the displacement fields (deflection, rotation, and interlayer slip). Then a finite element program is developed by assembling the local stiffness matrices and applying corresponding equivalent nodal stresses. Several numerical results are presented and compared against the analytical solutions available in the literature to demonstrate the accuracy of the proposed FE stiffness matrix. Finally, a design procedure by connecting particle swarm optimization technique with the present FE analysis is created to reduce the deformations of simply supported composite beams while the quantity of shear connectors remains the same, to prove the superior simulation capacity and efficiency of the derived FE stiffness matrix with other techniques. Compared to the analytical methods, the proposed finite element is more convenient and applicable in the analysis of partial-interaction composite beams under more complicated loading and boundary conditions. In the meantime, the explicitly expressed local stiffness matrix can be easily implemented into other commercial software packages as a subroutine for both professional and personal engineering designs and calculations.

1. Introduction

Steel-concrete composites combine the best attributes of their component materials. In this case, the high tensile strength and shear efficiency of steel and the compressive strength of concrete can be fully utilized. Experimental studies [1] show that interfacial slip between the concrete slab and the steel girder occurs even at low-level load, due to the finite rigidity of shear connectors. This phenomenon is called "partial interaction". Analytical studies have been carried out to investigate the static and dynamic behavior of composite beams with partial interactions based on Euler-Bernoulli beam theory coupled with interlayer slips [2–7]. In order to take into account of the effects of shear deformations, Xu and

* Corresponding author.
E-mail address: guannan.wang@ttu.edu (G. Wang).

http://dx.doi.org/10.1016/j.conbuildmat.2017.08.154
0950-0618/

Wu [8] used Timoshenko's beam theory together with consideration of interlayer slip to investigate the static, dynamic, and buckling behavior of partial interaction composite beams. Then Xu and Wang [9] formulated the minimum potential and complementary energy principles for composite beams, as well as the variational principles for the free vibration and buckling phenomenon, and proposed approximate solutions under different boundary conditions. Schnabl et al. [10] also gave the analytical solutions of a two-layer beam by taking into account of shear deformation. In addition, Xu and Wang [11] derived the relations of solutions of partial-interaction composite beams between Timoshenko and Euler-Bernoulli theories based on the constitutive equations.

Numerical methods, mainly based on the finite element method, have also been developed to investigate the partial-interaction composite beams [12–15]. What is more important, Thompson et al. [16] developed a finite element method for the analysis of layered wood systems. Ayoub and Filippou [17] derived an inelastic beam element from a two-field mixed formulation with independent approximation of internal forces and transverse displacements, and the partial interaction was accounted for by an interface model with distributed force transfer characteristics. Faella et al. [18] presented a displacement-based finite element model in which the stiffness matrix and the fixed-end nodal force vector are directly derived from the "exact" solution of Newmark's differential equation. Dall'Asta and Zona [19] proposed a three field mixed finite element for the non-linear analysis of composite beam with deformable shear connection. Čas et al. [20] presented a new finite element formulation by employing a modified principle of virtual work for the non-linear analysis of two-layer composite planar frames with an interlayer slip. In the finite element formulation the unknown functions are axial extensional strains of each layer and the pseudocurvature of the reference axis of the composite beam. Ranzi et al. [21] proposed an alternate formulation based on a direct stiffness approach that utilizes an internal solution for the slip in the same way as in Newmark et al.'s work [6]. Ranzi and Bradford [22] presented a stiffness formulation which is based on the direct stiffness method for the analysis of composite steel–concrete beam-columns with partial shear interaction. Ranzi and Zona [23] proposed an analytical model which was obtained by coupling the Euler–Bernoulli assumption for the reinforced concrete slab to the Timoshenko theory for the steel girder, for the analysis of steel–concrete composite beams with partial interaction including the shear deformability of the steel component. Valipour and Bradford [24] derived a flexibility-based element in the framework of the total secant approach for one dimensional composite elements with partial shear interaction. Jiang et al. [25] developed a two-node linear composite beam element for the steel–concrete composite beam with discrete shear connection. The element is derived by using the total potential energy method based on the Timoshenko beam theory and the linear Lagrangian interpolation function. Nguyen et al. [26] derived the exact stiffness matrix for a two-layer Timoshenko beam element with partial interaction basically inspired by the direct stiffness method. Martinelli et al. [27] presented closed-form analytical formulation of the stiffness matrix and the vector of equivalent nodal forces for analyzing shear-flexible steel-concrete composite beams in partial interaction. Brighenti and Bottoli [28] presented a finite element for the analysis of composite cross-section beams. The stiffness matrix of the finite element was obtained by using the direct stiffness method based on the theoretical solution of the problem provided in the literature. Taig and Ranzi [29] proposed a partial interaction formulation based on the generalized beam theory to study the partial shear interaction behavior of composite steel–concrete members. Other methods for the simulations of the partial interaction were using the interface elements to model the interfacial slips [30–32]. According to the authors' experience, numerical techniques (mainly FE method) are more applicable in broader engineering problems by involving efficient matrix implementations and avoiding tedious derivations.

Considered as an extension of the work by Xu and Wang [9], the current research employs Timoshenko's beam theory together with the consideration of the interlayer slip to derive the stiffness matrix of partial interaction composite beams. The present derivation distinguishes itself from other work in that it provides explicit expressions of the stiffness matrix of composite beams, which has not likely been provided previously. The stiffness matrix presented benefits not only professionals but also inexperienced designers or engineers for both qualitative learning and quantitative designs, without going through the process of physical inspirations and mathematical derivations. In addition, the explicit expressions can also be easily implemented into *ABAQUS* or *ANSYS* user subroutines as composite beam elements for more sophisticated structural analysis instead of building the elements from scratch. Finally, the explicit expressions can significantly improve the computational efficiency of the simulations, especially when it involves large-scale calculations or optimizations.

The organization of this work follows as: Section 2 provides the basic assumptions and formulations for the partial-interaction composite structures. The stiffness matrix is derived following the standard procedure based on the principle of minimum potential energy. The higher-order interpolation functions for the displacements are assumed. Detailed derivation of the stiffness matrix is provided in Section 3, and explicit expressions of stiffness matrix elements are presented in the Appendix A. A finite element program is then implemented based on the derivations. In Section 4, numerical results are also presented to illustrate the accuracy and efficiency of the proposed finite element method in the analysis of partial-interaction composite beams under complicated loading and boundary conditions. A new design procedure is introduced in Section 5 by employing particle swarm optimization technique connected with the FE program, aiming at reducing the deformations of composite beams and showing the advantage of the present FE stiffness matrix. Section 6 concludes this work.

2. Structural model formulation

2.1. Basic description and assumptions

The partial-interaction composite beam is composed of two sub-structures with different materials (steel-concrete) in the *x*-*z*

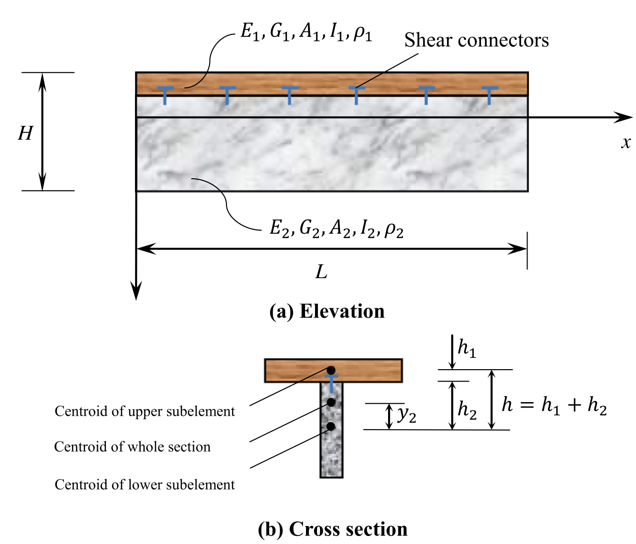

Fig. 1. Composite members and coordinate system: (a) elevation; (b) cross section.