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Development of master curve models complying with linear viscoelastic theory for complex moduli of asphalt mixtures with improved accuracy

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HIGHLIGHTS

- Established master curve models using approximate Kramers-Kronig relations.
- Demonstrated accuracy of master curve models constructed via proposed approaches.
- Verified compliance of constructed master curves with linear viscoelastic theory.
- Identified asymmetric shapes of master curves of four viscoelastic parameters.
- Proved nearly same time-temperature shift factors determined by both approaches.

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ABSTRACT

Approximate Kramers-Kronig relations have shown success in constructing master curves in compliance with the linear viscoelastic theory for the magnitude and phase angle of the complex modulus of an asphalt mixture. However, their applications have been limited to either shifting test data without model construction or constructing master curve models without addressing possible asymmetry of the master curves. Taking advantage of the Kramers-Kronig relations, this paper developed two approaches to establish the master curve models of four viscoelastic parameters of asphalt mixtures, including the magnitude and phase angle of the complex modulus as well as the storage modulus and loss modulus. In each approach, the four viscoelastic parameters shared the same time-temperature shift factor equation with exactly the same fitting parameters. Master curves of the viscoelastic parameters were constructed and were identified to have asymmetric shapes. These master curves were further verified to be compliant with the linear viscoelastic theory using black space diagrams and wicket plots, respectively.

It was found that the developed two approaches established master curve models with approximately the same accuracy level with R^2 values larger than 0.96. The two approaches also produced about the same time-temperature shift factor at each temperature. Therefore, either approach could be selected to accurately construct master curves of the four viscoelastic parameters in a wide frequency range.

1. Introduction

The complex modulus is an essential material property of an asphalt mixture in the linear viscoelastic state [1,2]. The complex modulus is usually denoted by E^* and is a complex function of the frequency:

$$E^*(\omega) = E' + iE'' \tag{1}$$

where ω = angular frequency, rad/s; E' = storage modulus, MPa; E'' = loss modulus, MPa; and i = imaginary unit. When investigating the damage evolution in the asphalt mixture, it is critical to construct the master curves of the magnitude ($|E^*|$) and phase angle (φ) of the complex modulus. These master curves provide the baseline properties for characterizing the departure of the asphalt mixture from the original linear viscoelastic state [3]. A variety of test methods have been developed to measure $|E^*|$ and φ at specific test temperatures, the test data of which have been used to construct the master curves of the complex modulus [4–8]. Most practices of the master curve construction can be grouped into the following types:

1. Constructing the master curve of $|E^*|$ using a master curve model with a shift factor equation, without constructing the master curve of φ [9–12];

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2. Constructing the master curves of both $|E^*|$ and φ utilizing two master curve models, respectively, which are independent of each other with either different shift factor equations or the same shift factor equation:
   (1) Different shift factor equations: the master curve models of $|E^*|$ and φ have different shift factor equations, and the parameters of the two shift factor equations are determined separately [7,8];
   (2) Same shift factor equation: the master curve models of $|E^*|$ and φ have the same shift factor equation with either different parameters or the same parameters:
      a. Different parameters: the parameters of the shift factor equation in the $|E^*|$ and φ master curve models are determined separately, which very likely result in a different set of parameters of the shift factor equation for the $|E^*|$ master curve from those of the shift factor equation for the φ master curve [13]; and
      b. Same parameters: the parameters of the shift factor equation are determined solely based on the test data of $|E^*|$ and are subsequently applied to the φ master curve construction; therefore, both $|E^*|$ master curve and φ master curve have the same shift factor equations with the same parameters [14,15].

The above practices of master curve construction may be able to produce smooth S-shaped master curves with high R^2 values of the model fitting. However, these practices are not compliant with the linear viscoelastic theory, whether constructing the $|E^*|$ master curve alone or constructing $|E^*|$ and φ master curves using two independent models, respectively. Since physical causality induces relationships between E' and E'' as well as between $|E^*|$ and φ in terms of the Hilbert transform, $|E^*|$, φ, E' and E'' are mathematically interrelated at any frequency [16,17]. As a result, these four viscoelastic parameters should share exactly the same shift factor equation with the same parameters, which should be determined based on the test data of both $|E^*|$ and φ (or the test data of both E' and E'') [17–21]. The relationships of the viscoelastic parameters are also known as the Kramers-Kronig relations or the Sokhotski–Plemelj theorem [16,17]. The exact Kramers-Kronig relations are presented in two sets of equations as follows, whose derivations are detailed in the literature [17,22,23]:

1. Set 1:

$$\ln|E^*(\omega)| = \frac{2}{\pi}\int_0^{+\infty}\frac{u\varphi(u)}{\omega^2-u^2}du \tag{2}$$

$$\varphi(\omega) = \frac{2\omega}{\pi}\int_0^{+\infty}\frac{\ln|E^*(u)|}{u^2-\omega^2}du \tag{3}$$

2. Set 2:

$$E'(\omega) = \frac{2}{\pi}\int_0^{+\infty}\frac{uE''(u)}{\omega^2-u^2}du \tag{4}$$

$$E''(\omega) = \frac{2\omega}{\pi}\int_0^{+\infty}\frac{E'(u)}{u^2-\omega^2}du \tag{5}$$

where: u = dummy variable in the range from 0 to infinity. Due to the challenges in solving the integrals in Eqs. (2) through (5), approximate Kramers-Kronig relations have been proposed as follows [22], which have been validated through experiments [22–26]:

$$\varphi(\omega) \approx \frac{\pi}{2}\frac{d\ln|E^*(\omega)|}{d\ln\omega} \tag{6}$$

$$E''(\omega) \approx \frac{\pi}{2}\frac{dE'(\omega)}{d\ln\omega} \tag{7}$$

Although the approximate Kramers-Kronig relations have shown success in constructing the master curves of $|E^*|$ and φ in compliance with the linear viscoelastic theory, their applications have been limited to:

1. Shifting test data without constructing the master curve model for either $|E^*|$ or φ [17–25]; or
2. Constructing master curve models of $|E^*|$ and φ without addressing possible asymmetry of the master curves [26].

These limitations are in fact obstacles for further tasks based on the master curve models:

1. Accurately predicting the values of $|E^*|$ and φ at frequencies out of the reduced frequency range of the shifted test data; and
2. Converting E^* into the relaxation modulus, $E(t)$, for the convenience of calculating the pseudostrain and the dissipated pseudostrain energy.

In order to eliminate such obstacles, this study focused on developing two approaches of constructing and validating the master curves of $|E^*|$ and φ of asphalt mixtures with appropriate fitting models complying with the linear viscoelastic theory. The next section presents the configurations and procedures of the laboratory experiments. The following section details the development of the two approaches of constructing the master curves of four viscoelastic parameters ($|E^*|$, φ, E' and E''). The subsequent section evaluates and compares the two approaches. The final section summarizes the major findings of this study and briefs the authors' ongoing research on this topic.

2. Laboratory experiments to determine $|E^*|$ and

2.1. Specimen fabrication

Two types of asphalt mixtures were prepared in this study for laboratory experiments. The asphalt mixture specimens were fabricated with the same type of limestone aggregates but two different asphalt binders, respectively, as shown in Table 1. The total air void content of each specimen was controlled at 4% ± 0.5%.

Three replicate specimens were fabricated and tested for every mixture type. The fabrication of the specimens included four steps that were implemented in sequence:

1. Heating and mixing: specific amounts of the asphalt, aggregate and fillers were heated in the oven and mixed in the mixer;
2. Compaction: after being cured for two hours, each batch of the loose mix was compacted into a cylindrical raw specimen with 150 mm in diameter and 170 mm in height using a Superpave Gyratory Compactor (SGC);
3. Coring and cutting: every raw specimen was cored and cut to 100 mm in diameter and 150 mm in height to achieve a more uniform air void distribution in the specimen; and
4. Gluing: three sets of brackets were glued 120° apart from each other to the side surface of each specimen to hold the linear variable differential transformers (LVDTs).

2.2. Test procedure

A Dynamic Testing System (DTS) was employed to measure the $|E^*|$ and φ of each fabricated specimen. This DTS had a servo-hydraulic actuator of 30 kN with a stroke of 100 mm and was equipped with an environmental chamber capable of precisely controlling temperature. During the test, each specimen was set up in the environmental chamber with end platens placed at the