Construction and Building Materials 152 (2017) 310–318

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Vertical stiffness degradation of laminated rubber bearings under lateral deformation

Weiguo Yang, Xinyang Sun *, Meng Wang, Pei Liu

School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China

HIGHLIGHTS

- A vertical stiffness degradation formula of elastomeric bearing is proposed.
- Bearings with different cross-sectional shapes have same vertical stiffness ratio.
- Vertical stiffness degrades to half initial stiffness at limit lateral deformation.
- Vertical stiffness ratio is unaffected by vertical loading direction.

ARTICLE INFO

Article history:
Received 5 January 2017
Received in revised form 23 May 2017
Accepted 1 July 2017

Keywords:
Laminated rubber bearing
Vertical stiffness
Seismic isolation
Two-spring model
Lateral deformation
Finite element analysis

ABSTRACT

In order to investigate the vertical stiffness degradation of laminated rubber bearing, which is a widely used composite of steel and rubber in structures, the vertical stiffness ratio formulas were presented based on the two-spring model and overlapping area model. Then the details of finite element models of bearings were introduced, which were proved to be able to predict the bearings behavior by comparing with typical experimental results. The regression curve of vertical stiffness ratio deduced based on 3D finite element analysis of bearings with various section shapes and sizes was compared to results from theoretical models and experimental approaches. The results showed that the vertical stiffness ratio is only determined by the ratio of the lateral deformation to inertia radius of section, and is unaffected by the loading direction (compression or tension), section shape and size of bearings. The current models underestimate the vertical stiffness ratio of bearings, whereas the regression curve proposed in this study showing better agreement compared to theoretical and experimental results.

1. Introduction

Seismic isolation is an effective method to reduce the inertial forces of structures under earthquakes by lengthening the fundamental period, as well as capable of carrying large vertical loads [1,2]. Thus, it has been extensively used in buildings, bridges and other important structures over the last three decades [3,4]. Laminated rubber bearing is a commonly used seismic isolation device. Previous analyses have shown that seismic isolation bearings can reduce stresses and accelerations in the superstructure, and the horizontal displacements are mainly generated in the isolation layer [5]. During the severe earthquakes, these bearings will be subjected to large rubber shear strain, usually more than 100%, which changes the lateral behavior of bearings [6], furthermore, accompanied by the degradation of vertical stiffness. However, the vertical stiffness of the bearing is still simplified as a linear spring in seismic codes and numerical earthquake simulation software, which seriously affects the safety of the isolation system and the superstructure [7].

In order to study the degradation behavior of seismic laminated rubber bearings, researchers around the world conducted a wide range of analyses, many mathematical models were established and corresponding calculation formulas of the vertical stiffness of the isolation rubber bearing were derived, based on the linear assumption and small deformation theory. Gent [8] and Lindley [9] had put forward the formula to calculate the vertical stiffness for bearings, treating the vertical stiffness as linear and independent with shear deformation. After that, Koh and Kelly [10] proposed a two-spring model to analyze the influence of axial load on the height reduction, horizontal stiffness, as well the damping of bearings. Buckle and Liu [11] established a more simplified overlapping area model to estimate the critical load of bearings under the application of combined lateral displacement and compression.

* Corresponding author.
E-mail addresses: wgyang1@bjtu.edu.cn (W. Yang), 13115301@bjtu.edu.cn (X. Sun), wangmeng@bjtu.edu.cn (M. Wang), peiliu@bjtu.edu.cn (P. Liu).

http://dx.doi.org/10.1016/j.conbuildmat.2017.07.004
0950-0618/

The two-spring model and overlapping area model were later applied by Warn [7] to investigate the effect of shear deformation on compression stiffness degradation of annular bearings. Further, based on the experimental results presented by Sanchez [12], the coupled horizontal and vertical behavior of bearing was studied for dynamic loading by Vemuru [13]. Moreover, Kelly [14–16] considered the lateral displacement in tension bulking of elastomeric bearings, which might be happened unexpectedly in base-isolated tall buildings, summarized the mechanical relation of bearing in tension and compression. The above researches have significant effects on analyzing the vertical stiffness theory, and reducing the adverse impacts of vertical stiffness degradation of bearings on structures.

However, most of these studies focused on one or a few section shapes of bearings, or only for the compression stiffness, there is no systematic and comprehensive comparison of the vertical stiffness degradation of rubber bearings with different cross section shapes and different sizes. Therefore, in this study, firstly, according to the two-spring model and overlapping area model, theoretical formulas of vertical stiffness degradation of four section shapes of bearings (circular, annular, square and rectangular) were derived. To evaluate these formulas, seven finite element models of laminated rubber bearing were established using ABAQUS software. It was proved that the models are able to accurately simulate the actual behavior by comparing with typical experimental results. The details of the model such as constitutive model and meshing were also illustrated. Then, based on the verified numerical method, the vertical stiffness degradation rules were deeply analyzed, and the simulation results were compared with theoretical formulas. Finally, a normalized vertical stiffness degradation formula which is suitable for different section shapes, sizes and load directions was proposed. It provides a powerful tool for seismic response analysis of isolated structures.

2. Theoretical models for vertical stiffness degradation

2.1. Two-spring model in compression direction

Considering the shear and flexural deformation under the action of gravity of upper structure and seismic load, the rubber bearing is normally modeled as a Haringx column [17]. By taking the *P*-*Δ* effect into account, Koh and Kelly [10] developed the two-spring model for combined compression and shear, which is simplified with only shear stiffness and rotational stiffness degree-of-freedom. The un-deformed and deformed illustration for compression is shown in Fig. 1.

The model contains an infinite-stiffness column with length of l, which is a known parameter in the model, a horizontal and a rotational spring with the stiffness of K_H and K_θ, respectively. The shear force F and compression force P are loaded simultaneously at the peak of the column, resulting the rotation of the column θ, linear spring deformation s, height reduction v, and horizontal displacement u. The v including two parts, namely, the applications of compression force, v_c, and the component resulted from combined compression and shear, v_s. The deformation u and v_s can be calculated according to Eqs. (1) and (2):

$$u = s\cos(\theta) + l\sin(\theta) \tag{1}$$

$$v_s = s\sin(\theta) + l[1 - \cos(\theta)] \tag{2}$$

Due to the actual rotation θ is small enough, here sin θ could be simplified as θ. By approximating cos θ with its first two terms of Taylor series expansion, cos θ could be expressed as $1 - \theta^2/2$. Then the deformation u and v_s can be calculated using Eqs. (3) and (4):

$$u = s + l\theta \tag{3}$$

$$v_s = s\theta + \frac{l\theta^2}{2} \tag{4}$$

Meanwhile, considering the equilibrium of model under compression and shear force, the following equation can be obtained:

$$\begin{bmatrix} P_E - P & -P \\ -P & GA_s \end{bmatrix} \begin{Bmatrix} \theta \\ \frac{s}{l} \end{Bmatrix} = \begin{Bmatrix} F \\ F \end{Bmatrix} \tag{5}$$

where $K_\theta = P_E l$, $A_s = A_b l/T_r$, $P_E = \pi^2 EI_s/l^2$, $I_s = Il/T_r$, $E = E_c/3$, P_E is Euler buckling load, A_b is rubber section area, T_r is total thickness of rubber, E_c and G are compression modulus and shear modulus of rubber, respectively, A_s and I_s are the sectional area and moment of inertia of bearing, respectively, which accounting for the steel plates. By assuming the steel layers as rigid compared with rubber, and bringing the factor "l/T_r", the bearing could be regarded as a homogeneous column. Substituting the calculated horizontal deformation s and rotational angel θ from Eq. (5) into Eqs. (3) and (4), besides, considering that for laminated rubber bearings, it is known that $GA_s << P_E$ and $P << P_E$ [7,18], the height reduction v_s could be obtained as Eq. (6):

$$v_s = \frac{3(GA_b l + PT_r)u^2}{\pi^2 E_c I} \tag{6}$$

Additionally, the first part of height reduction of bearing is $v_c = PT_r/E_cA_b$, so the total reduction is given by:

$$v = \frac{3(GA_b l + PT_r)u^2}{\pi^2 E_c I} + \frac{PT_r}{E_c A_b} \tag{7}$$

The above formulation is the total vertical deformation of bearings under combined horizontal and vertical load, which is in accordance with the results from Warn [7].

As the compression stiffness of bearing under the axial load is $K_v = P/v$, so the compression stiffness could be expressed by lateral deformation, as Eq. (8):

$$K_v = \frac{E_c A_b}{T_r} \frac{1}{1 + \frac{3A_b u^2}{\pi^2 I}\left(1 + \frac{GA_s}{P}\right)} \tag{8}$$

Since $GA_s < P$ for bearings, and the influence of GA_s/P on compression stiffness ratio is small enough [19], so this term could be neglected in Eq. (8). Setting the lateral deformation $u = 0$, the Eq. (8) reduces to the initial compression stiffness, which is $K_{v0} = E_cA_b/T_r$. So the compression stiffness ratio of bearing K_v/K_{v0} is calculated according to Eq. (9):

$$\frac{K_v}{K_{v0}} = \frac{1}{1 + \frac{3A_b u^2}{\pi^2 I}} \tag{9}$$

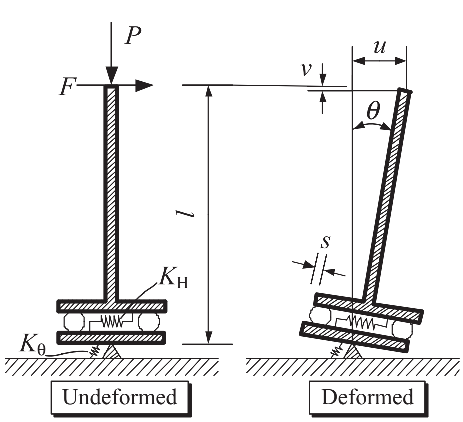

Fig. 1. Two-spring compression model.