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Quasi-analytical homogenization approach for the non-linear analysis of in-plane loaded masonry panels

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HIGHLIGHTS

- Quasi analytical holonomic homogenization model for masonry in-plane loaded.
- Subdivision of the elementary cell into elastic triangles (bricks) and non-linear interfaces (mortar).
- Comprehensive validation at a cell level in the elastic and inelastic range.
- Double structural implementation: nested multi-scale FE2 and rigid body and spring model RBSM.
- Validation of a windowed shear wall against experimental data and previous numerical models.

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ABSTRACT

A simple holonomic compatible homogenization approach for the non-linear analysis of masonry walls in-plane loaded is presented.

The elementary cell (REV) is discretized with 24 triangular elastic constant stress elements (bricks) and non-linear interfaces (mortar). A holonomic behavior with softening is assumed for mortar joints. It is shown how the mechanical problem in the unit cell is characterized by very few displacement variables and how the homogenized stress-strain behaviour can be evaluated semi-analytically. At a structural level, it is therefore not necessary to solve a FE homogenization problem at each load step in each Gauss point.

Non-linear structural analyses are carried out on a windowed shear wall, for which experimental and numerical data are available in the literature, with the aim of showing how quite reliable results may be obtained with a limited computational effort.

1. Introduction

Masonry is a traditional composite material obtained by the assemblage of bricks and mortar. The variability of the pattern, the shape and dimension of the blocks, as well as the fragile behavior of the constituent materials, make the simulation of masonry still a very challenging task. The elastic behavior is quite limited because masonry is typically characterized by a reduced, almost vanishing tensile strength. Therefore, numerical models traditionally exhibit a moderate level of complexity, because they are native non-linear. As a matter of fact, either macro- or micro-modeling strategies are adopted to deal with masonry over elasticity.

Macro-modeling substitutes bricks and mortar with a homogeneous, sometimes orthotropic material with softening. Abundant is the literature in this regard, see for instance [1–3], with the noticeable example of no-tension material modeling (e.g. [1]), which traditionally was conceived to deal with non-linear problems exhibiting predominant mode I fracture of the joints (e.g. arches or pillars under rocking) and masonries with very good compressive strength, where crushing and orthotropic behavior are not paramount. Macro-modeling allows studying even large scale structures without the need of meshing separately bricks and mortar. It is therefore very convenient where efficient computations on engineering structures are needed. Nevertheless, the calibration of model parameters is typically done by means of comprehensive experimental campaigns. When the level of sophistication of the model increases [2,3], to better reproduce anisotropy, post-peak softening in tension and compression and a Mohr-Coulomb shear behavior with compression cap, the number of inelastic parameters grows and the experimental characterization may become costly and cumbersome. Theoretically, such approaches may be

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capable of adequately estimating the non-linear masonry behavior for an arbitrary load combination, even if some meaningful limitations occur in specific cases (see [4] for a detailed discussion), but in practice the needed experimental data fitting would require – at least in principle- new calibrations case by case.

The alternative micro-modeling is simply characterized by a distinct modeling of mortar joints and blocks at a structural level. The reduction of joints to interfaces [5–9] helps in limiting variables, especially in the non-linear range, but the approach still remains computationally very demanding, because bricks and mortar are meshed separately. In order to obtain sufficiently reliable solutions in terms of displacements and stresses, constituent materials should be meshed with more than one element, with the consequent grow of the number of non-linear equations to deal with, even for small masonry panels. Furthermore, the preprocessing phase regarding the model generation is not straightforward. Partitioning methods have been recently proposed to overcome such computational limitations and speed up structural analyses.

For the previous reasons, it can be affirmed that macro-scale computations with FEs [10,11] still remain preferable when non-linear analyses for engineering structures are needed.

In such a scenario, homogenization [12–23] may represent a fair compromise between micro- and macro-modeling, because it allows in principle to perform non-linear analyses of engineering interest without a distinct representation of bricks and mortar, but still considering their mechanical properties and the actual pattern at a cell level.

Homogenization (or related simplified approaches) is essentially an averaging procedure performed at a meso-scale on a representative element of volume (REV), which generates the masonry pattern under consideration by repetition.

On the REV, a Boundary Value Problem BVP is formulated, allowing an estimation of the expected average masonry behavior to be used at structural level. As a matter of fact, the resultant material obtained from meso-scale homogenization turns out to be orthotropic, with softening in both tension and compression. A straightforward approach to solve BVPs at the meso-scale is obviously based on FEs [15,20–23], where bricks and mortar are either elasto-plastic with softening or damaging materials. It is also known as FE^2 and essentially is a twofold discretization, the first for the unit cell and the second at structural level. However, FE^2 appears still rather demanding, because a new BVP has to be solved numerically for each load step, in each Gauss point.

Alternatively, in this paper, a simplified homogenization two-step model is proposed for the non-linear structural analysis of masonry walls in-plane loaded. The first step is applied at the meso-scale, where the assemblage of bricks and mortar in the REV is substituted with a macroscopic equivalent material through a so called compatible identification, belonging to the wide family of the homogenization procedures. The unit cell is meshed by means of 24 triangular constant stress (CST) plane stress elements (bricks) and interfaces for mortar joints. Triangular elements are assumed linear elastic, whereas the mechanical response of the interface elements includes two dominant deformation modes, namely peel (mode I) and shear (mode II) or a combination of two (mixed mode). Such elements are equipped with a constitutive relationship referred to as "holonomic" since expressed in terms of normal and tangential tractions σ and τ as a path independent function of the normal and tangential relative displacements at the interface. Both a piecewise linear and an exponential law formally identical to an improved version of the Xu-Needleman law and proposed in another context [24–26] are implemented. Such cohesive relationships are characterized by a post-peak softening branch, eventually with a coupling between normal and shear relationships in the case of the improved Xu-Needleman model.

The second step, performed at a structural level, relies into the implementation of the homogenized stress-strain relationships into either a FE code dealing with softening materials (nested multi-scale technique) or a rigid element approach (RBSM) where contiguous rigid elements are connected by shear and normal non-linear homogenized springs.

The first approach (nested multi-scale technique) is very similar to FE^2, but has the advantage that the BVP at the meso-scale level is solved in quasi-analytical form. Limitations of FE^2 are therefore totally superseded, since the solution in terms of displacements and stresses is found at a cell level in a semi-analytical fashion, with an implementation of the routine used at a meso-level to evaluate homogenized quantities directly at a structural level. As a consequence, the scale passage does not require the huge computational effort needed by FE^2.

The second approach (RBSM) has the advantage that meso- and macro-scale are fully decoupled, i.e. homogenized stress-strain non-linear relationships of the springs connecting rigid elements are evaluated in a previous phase, without the need of solving new BVPs at each load step in each Gauss point. The disadvantage of RBSM is the intrinsic mesh dependence of the results in case of global softening.

In both cases, it is worth mentioning that any commercial code can be suitably used for the implementation of the homogenization model proposed.

The procedure is quite efficient and reliable because it is not necessary to discretize with refined meshes the elementary cell (only three kinematic variables are needed at the meso-scale) and hence it is possible to drastically speed up computations. In addition, the holonomic laws assumed for mortar allow for a total displacement formulation of the model, where the only variables entering into the homogenization problem are represented by displacements.

Notation: Vectors and tensors are indicated in bold. $\mathbf{E}$ and $\mathbf{\Sigma}$ indicate strain and stress homogenized tensors, x (y) is the horizontal (vertical) in plane direction, E_{xx} ($E_{yy}, E_{xy} = \Gamma_{xy}/2$, E_{nn}) is the macroscopic horizontal (vertical, shear, on direction n) strain, Σ_{xx} ($\Sigma_{yy}, T_{\mathrm{hom}}$) is the homogenized horizontal (vertical, shear) stress, $\sigma_{xx}^{(k)}$ ($\sigma_{yy}^{(k)}, \tau^{(k)}$) is the local horizontal (vertical, shear) stress on element k, $\varepsilon_{xx}^{(k)}$ ($\varepsilon_{yy}^{(k)}, \varepsilon_{xy}^{(k)} = \gamma_{xy}^{(k)}/2$) is the local horizontal (vertical, shear) strain on element k, L (H) is the brick semi-length (height), $\rho = L/2H$, A is the elementary cell (REV) area, 2 e_v (e_h, e) is head (bed, generic) joint thickness, U_x^0 (U_y^0) indicate an imposed boundary horizontal (vertical) displacement in the biaxial strain problem, U_x^i (U_y^i) is the i-th node unknown horizontal (vertical) displacement, Δ_n (Δ_t) is the interface normal (tangential) jump of displacements, $f_n^{I,II}$ ($f_t^{I,II}$) is the joint (I: head, II: bed) normal (shear) stress, $\xi = U_x^0 - U_x^9$, $\eta = U_y^5 + U_y^6$, E_b (ν_b, G_b) is the brick Young modulus (Poisson's ratio, shear modulus), E_m (G_m) is mortar Young (shear) modulus, D_{ijhk} is the homogenized elastic stiffness $ijhk$ component, Δ_n^{ul} (Δ_t^{ul}) is the ultimate joint normal (tangential) jump of displacements in the multi-linear model, f_t (c) is joint tensile strength (cohesion), δ_n δ_t ϕ_n and ϕ_t are Xu-Needleman interface parameters, $\xi^t = U_x^5 + U_x^6$, $\eta^t = U_y^3$, $\kappa = \tau^{(1)} e_v / G_b$, $\bar{U}_x^t$ ($\bar{U}_y^t$) indicate an imposed boundary horizontal (vertical) displacement in the shear problem, $\vartheta = \tan^{-1}(E_{yy}/E_{xx})$.

2. The simplified (compatible homogenization) holonomic model

One of the basic concepts of homogenization is the utilization of averaged quantities for the macroscopic strain and stress tensors (respectively $\mathbf{E}$ and $\mathbf{\Sigma}$) [15,20–22,27] on a representative element