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Ultimate strength of horizontally curved steel I-girders with equal end moments

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ABSTRACT

Even under gravitational loading alone, horizontally curved girders experience not only bending moment but also torsional moment. The torsional moment acting on open sections simultaneously produces shear and normal stress due to pure and warping torsion respectively. Consequently, bending moment, pure torsion and warping torsion are coupled, which results in a very complicated stress state that makes it difficult to calculate the ultimate strength of horizontally curved members. This study revealed that the initial curvature can reduce the ultimate strength of horizontally curved members by up to 50%. Although current design specifications such as the AASHTO LRFD Bridge Design Specifications, suggest some alternatives, the exact behavior of a curved member cannot be considered well with those provisions. While it is true that the one-third rule is convenient to apply and gives good results, there is no strength equation for curved members. In order to derive an adequate strength equation for curved members, this research suggests a new concept of ultimate state. Finite element analysis using ABAQUS is used to consider the effects of sectional rigidities for bending, pure torsion and non-uniform torsion separately. Finally, an ultimate strength equation is suggested for simply supported curved girders that are subjected to equal end moments.

1. Introduction

1.1. Background

As a structural beam member, the I-girder is subjected to bending moment. When equal end moments are applied to a simply supported straight I-girder (pure moment condition), compression flanges tend to show buckling behavior, but are restrained by their continuous connection to the web. Consequently, the entire section experiences out-of-plane and rotational displacement, which is typically referred to as lateral torsional buckling. If local instability is prevented, the lateral torsional buckling strength can govern the ultimate strength.

On the other hand, when an I-shaped girder is subjected to torsional moment, shear and normal stresses are produced by pure torsion and warping torsion respectively. If bending moment acts at the same time, the stress components are coupled and the stress state becomes very complicated. Straight girders, however, are seldom subjected to large torsional moment hence the moment-torsion interaction does not need to be significantly considered in design or analysis. It is reasonable to assume that rotational displacement in straight girders occurs only if lateral torsional buckling occurs.

In the case of a horizontally curved girder, bending and torsional moments always occur simultaneously due to the curvature effect. Because of the torsional moment, lateral and rotational displacements are inevitable responses of curved I-girders. In other words, design of horizontally curved girders should consider moment-torsion interaction.

Despite the difficulties presented by the coupling of bending and torsional moment, there are very few design specifications for curved members. Even the 2014 AASHTO LRFD bridges design specifications [1], which includes some provisions for horizontally curved members, does not provide any countermeasure for the ultimate strength of I-girders curved in plan that considers moment-torsion interaction.

In this research, in order to investigate the ultimate strength of horizontally curved girders, ABAQUS 6.13 was used to perform finite element analyses. Geometrical and material nonlinearity were considered in the analyses, so that the ultimate strength of the curved member was estimated. All the models were subjected

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to equal end moments (which means the moment gradient factor C_b is 1.0 for a straight girder) and each result for curved members was compared with those of straight girders. Through the analyses, the behavior and strength of a curved member could be investigated.

1.2. Previous researches

There have been many researches that deal with the behavior of horizontally curved girders. Fukumoto et al. [2] introduced the transfer matrix method to investigate the behavior of curved members under large torsional deflection. Subsequently, Yoshida and Maegawa [3] evaluated the ultimate strength of a curved girder. However, they assumed that the material yields due to normal stress only; i.e., the effect of the torsional moment was neglected. Yang and Kuo [4] evaluated the curvature effect using the principle of virtual work.

Yoo and Pfeiffer [5] presented a method for the elastic torsional buckling of curved girders by correcting Vlasov's theory that had been adopted for torsional behavior. A set of design charts for curved beams was developed from their studies.

There have been many efforts to develop a curved beam element. After Dabrowski [6] utilized the differential equation of a curved beam, Morris (1968), Ho (1972), El-Amin (1976) and Hsu (1990) [7–10] formulated curved beam elements. Finally Kang [11] and Kang and Yoo [12] formulated the curved beam element based upon thin-walled curved beam theory and implemented it into a computer program.

Meanwhile, several researches have also been conducted into ultimate load carrying capacity and moment-torsion interaction. Liew et al. [13] conducted combined geometric and material nonlinear analyses that considered residual stresses for curved I-girders and compared the analysis results with the results from experimental studies. They modified Fukumoto's method [2] to predict the ultimate strength of curved beams. The high order term was included in order to evaluate a more accurate solution. Finally, they suggested a strength equation for curved beams. However, their method was quite complicated and inaccurate.

Experimental studies involving curved girders have also been carried out. Shanmugam et al. [14] investigated the behavior of curved steel I-beams with various radii of curvature. Their analytical results demonstrated good agreement with experimental studies and showed the relationship between the curvature and load carrying capacity of curved members. In their research, the main factor related to the ultimate strength was focused not on the moment-torsion interaction, but on residual stresses; and a general strength equation was not discussed.

Pi and Trahair [15] performed an analytical study with the intention of verifying the nonlinear elastic behavior of I-beams curved in plan. After that, Pi and Bradford [16] further studied fully nonlinear behaviors. In accordance with the two papers, the behavior of curved girders was classified according to the subtended angle. The strength of curved beams with a small subtended angle ($\theta \leq 1°$) was dominated by bending action so it was concluded that they could be regarded as straight girders. For the girders that have a subtended angle of $1° < \theta < 20°$, both lateral and rotational displacements become large. As the subtended angle increases, the significance of bending action decreases for the beams that have a large subtended angle ($\theta > 20°$) torsion becomes a major action. With these results, Pi and Bradford [17] suggested a moment-torsion interaction curve as follows:

$$\left(\frac{M_u}{M_{bx}}\right)^{\gamma x} + \left(\frac{T_u}{T_P}\right)^{\gamma_s} \leqslant 1 \quad (1)$$

where

M_u: maximum nominal in-plane bending moment determined by first-order elastic analysis
T_u: maximum nominal torque determined by first-order elastic analysis $\gamma_x \& \gamma_s$
For continuously braced curved beams; $\gamma_x = 2.0; \gamma_s = 1.0$
For centrally braced curved beams; $\gamma_x = 1.5; \gamma_s = 1.0$
For unbraced curved beams; $\gamma_x = 1.0; \gamma_s = 1.0$
M_{bx}: Nominal member moment capacity of corresponding straight beam with the same length and boundary conditions
T_P: Full plastic torque for nonuniform torsion.

Eq. (1) provides a very convenient way to consider bending and torsional moment separately. However, it leads to conservative results for sharply curved girders (i.e., the girder subjected to large amount of $M_{cr(cu)} = \sqrt{1 - \frac{L^2}{\pi^2 R^2}}\sqrt{(\frac{\pi^2 EI_y}{L^2})(GJ + \frac{\pi^2 EC_w}{L^2})}$ torsional moment).

For the lateral torsional buckling strength of girders curved in plan, two representative researches can be referenced. Nishida et al. [18] suggested the following equation for the elastic lateral torsional buckling strength of curved beams. (Eq. (2))

$$M_{cr(cu)} = \sqrt{1 - \frac{L^2}{\pi^2 R^2}}\sqrt{\left(\frac{\pi^2 EI_y}{L^2}\right)\left(GJ + \frac{\pi^2 EC_w}{L^2}\right)} \quad (2)$$

where

$M_{cr(cu)}$: Elastic buckling strength for LTB of curved girder with equal end moment
L: Length of the girder
R: Radius of curvature
G: Shear Modulus
J: Torsional Constant
C_w: Warping Constant.

With this equation, the curvature effect can be easily considered as a kind of strength reduction factor that is multiplied by the strength of the straight girder. However, the term of L^2/R^2 represents the square of the subtended angle θ, and therefore the sectional properties are not involved in the curvature effect prediction. Moreover, this equation is only applicable to linear elastic analysis.

In 1996, Yoo et al. [19] carried out a set of parametrical studies using a 7-D.O.F element that was developed by Kang and Yoo [12]. Through this study, the ultimate strength equation of a horizontally curved girder was suggested. The equation provides a strength reduction factor that is a function of the radius of curvature, and can be applied not only to elastic torsional buckling moment (critical moment) but also to the ultimate bending moment of a curved girder. Eq. (3) Yoo and Davidson [20] developed this equation with yield interaction equations. Subsequently, the authors (Davidson and Yoo [21]) conducted an analytical study using a commercial finite element program, and compared the results with previous strength predictors.

$$[M_P, M_u, M_{xcr}]_{cu} = (1 - 0.1058\theta^{2.129})^{2.152}[M_P, M_u, M_{xcr}]_{st} \quad (3)$$

Where

$M_{p)cv}$: Plastic bending moment of the section of a curved beam
$M_{u)cv}$: Ultimate bending moment of the section of a curved beam
$M_{xcr)cv}$: Lateral-torsional buckling moment of a curved beam
M_p: Plastic bending moment of the section of a straight beam
M_u: Ultimate bending moment of the section of a straight beam
M_{xcr}: Lateral-torsional buckling moment of a straight beam.

Similar to Nishida's results [18], however, only curvature is considered with this equation. Furthermore, although it was evaluated