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An experimentally validated finite element formulation for modeling 3D rotational energy harvesters

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ABSTRACT

Piezoelectric energy harvesting devices convert mechanical energy into electrical energy due to the mechanical deformations of the structures. Energy harvesting prototypes are used to feed low-power electronic devices and sensors. In this work, a one-dimensional finite element is developed for modeling three-dimensional rotational energy harvesters. The rotating piezoelectric beam is formulated by means of a geometrically nonlinear finite element with six mechanical degrees of freedom and one electrical degree of freedom per node. Using Timoshenko beam theory for the mechanical domain and a first-order theory for the electrical field, the electromechanical equilibrium equations of motion are then derived using D'Alembert principle. In order to validate our finite element formulation, two energy harvesting devices are built and tested, getting insights into the generation of electrical power, natural frequencies and time responses of the dynamical variables. An Arduino board is implemented as the data acquisition system that transfers the voltage signal via Bluetooth, avoiding the complexity of slip-rings mechanisms for data transmission. Finally, the results of our formulation are compared with those obtained using a commercial software (Abaqus) and the experimental results. A good correlation between the three methods is obtained, providing evidence that our formulation accurately predicts the behavior of rotational energy harvesters.

1. Introduction

The dynamic behavior of rotating structures has been studied for many years. There are numerous reports in the literature that analyze the vibration of rotating beams. Carnegie [1] investigated the vibration of rotating cantilever blading, obtaining a theoretical expression for the work done due to centrifugal and Coriolis effects. Boyce and Handelman [2] studied the transverse vibration and the influence of a tip mass placed at the free end of a cantilever beam taking into account a constant speed. Hoa [3] proposed a finite element (FE) method to investigate the vibration frequency of a rotating cantilever beam with a tip mass. The finite element method was based on a third-order polynomial for the variation of the lateral displacement. Geradin and Kill [4] developed a new approach to finite element modeling applied to flexible rotors in order to perform the stability analysis. Their models were developed with the rotating frame and the inertial frame approaches. In both cases an asymmetric finite element is proposed. Aircraft wings and blades are applications of the investigations about the dynamic behavior of rotating structures. Recently, composite materials have been widely used in the main structure for increasing the performance of the blades or wings. Saravia et al. [5] investigated the dynamic stability behavior of thin-walled rotating composite beams using the finite element method. Due to the flexibility of composite structures, control of the vibrations is essential [6]. For this reason, in the last decades the inclusion of smart materials in the main structure has been studied. The suppression of vibrations is improved using active control in the structures with piezoelectric actuators. Piezoelectric materials are usually of interest when designing smart structures that can be used as sensors or actuators [7,8]. In the last few years, energy harvesting has received increasing attention due to its applications. It converts the waste energy into usable electrical energy, or in other words, mechanical vibrations are converted into electrical energy used to power mobile devices and wireless sensors networks [9]. Many researchers have derived mathematical models for energy harvesting beams [10]; most of them have used the Euler–Bernoulli beam theory with a harmonic base excitation. Erturk and Inman [11,12] studied a distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters. Mitcheson et al. [13] presented a state-of-

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art review of energy harvesting devices including possible applications and future developments.

Finite element plate models are also used to investigate piezoelectric structures under base harmonic motion. Marqui Junior et al. [14] proposed an electromechanical coupled finite element plate model for predicting the electrical power output of piezoelectric energy harvester plates. The excitation is due to the harmonic motion of the base in the transverse direction. Detwiler et al. [15] formulated a laminated composite plate with piezoelectric material to analyze the mechanical-electrical behavior. The top and bottom surface of the beam were subjected to an electric potential of 1 V across the thickness of the beam, and the corresponding displacements were determined. On the other hand, there are several finite element software packages (such as Abaqus or ANSYS) for the modeling of piezoelectric materials. Kumar et al. [16] analyzed the performance of lead-free piezoelectric materials in unimorph cantilever piezoelectric energy harvesters. The finite element method was used to model the piezoelectric structure, while genetic algorithm optimization was used to optimize the power output. Staworko and Uhl [17] presented an overview of modeling techniques of piezoelectric elements and their comparison with commercial software for simulating electromechanical systems. The models built using Simulink and PSPICE give a slight difference of the frequency of vibration with respect to Ansys simulation, whereas the amplitudes of the time response using Simulink and PSPICE are three times larger than those obtained with Ansys. Elvin et al. [18] developed a coupled finite element-circuit simulation model using finite element software packages. For the mechanical domain, a finite element method was used to calculate the dynamic response. In the electronic domain, the simulation tool SPICE was used to calculate the electrical response. This approach allows for the modeling of complex mechanical geometries. However, the solution technique is computationally expensive for large models. Zhou et al. [19] proposed an equivalent SDOF system to describe the energy harvesting performance of a cantilever beam. The peak output power and voltage were compared with numerical simulation using the commercial FE package Ansys.

In the last few years a number of investigations have been published about rotational energy harvesters. Gu and Livernore [20] presented a single-degree-of-freedom (SDOF) model taking into account the centrifugal force of the tip mass of a passive self-tuning energy harvester for rotational vibration applications. Khameneifar et al. [21,22] also presented an analytical model with a SDOF considering the centrifugal force of the tip mass and the gravity force of the whole model. Guan and Liao [23] developed a novel design of a rotating harvesting structure using an analytical model that assumes the whole system mass in the centrifugal and gravity forces. They analyzed the device theoretically and experimentally. Their results do not predict with sufficient accuracy the experimental results. Shahruz and Sundararajan [24] proposed a SDOF mathematical model of a cantilever beam with a tip mass considering the whole system mass in centrifugal and gravity forces. They provided a guideline for the scavenger parameters in order to have it resonate. Yang et al. [25] investigated experimentally an improved the output power of a rotational piezoelectric wind energy harvester. They proposed an impact force to enable effective excitation. On the other hand, Hsu et al. [26] used a finite element software package (COMSOL) to simulate a rotating cantilever beam with a tip mass. They analyzed self-frequency tuning piezoelectric energy harvesters for rotational motion. Their FE model takes into account the shear deformation, piezoelectric effect, and stress stiffening effect induced by the centrifugal forces of the entire mass. Their results were compared with an analytical model and experimental tests.

In the above references the analysis of energy harvesting devices was limited to simple geometries such as cantilever beams [20–22]. In Refs. [23,24] the centrifugal and gravitational forces applied to the SDOF system were considered in the FE model, but the softening effect induced by the rotation speed was neglected. In [26] Hsu et al. developed a FE model using a commercial software (COMSOL) based on three dimensional (3D) solid elements to model the rotational energy harvester. This approach allows to solve the difficulties mentioned above, but it has a large computational cost due to the 3D elements. In the present work, in contrast to current scientific literature, a one-dimensional finite element is developed to model 3D rotational energy harvesting devices. Within this approach it is possible to model complex geometric configurations, the geometrically nonlinear effect induced by the centrifugal forces and the electromechanical coupling. The piezoelectric beam is formulated by linearizing a geometrically nonlinear FE with six mechanical degrees of freedom per node and one electrical degree of freedom interpolated using standard linear shape functions. Timoshenko beam theory is used for the mechanical domain [5], and a first order theory is used for the electrical domain [31].

The present article is organized as follows. After the introduction, the kinematics of a piezoelectric rotating beam is presented in Section 2. Section 3 presents the details of the variational principle to derive the equilibrium equations of motion of the problem using D'Alembert principle. Section 4 presents the formulation of the piezoelectric beam element. Section 5 contains the experimental setup to validate our FE approach. Section 6 shows a comparison of the natural frequencies between our FE formulation and the numerical simulations using Abaqus. The voltage time response and the voltage and power generation of our proposed model are also compared with the experimental results. Finally, Section 7 presents the conclusions.

2. Kinematics

The main aspects of the present rotational energy harvester formulation are the following:

- The kinematics is based on the Timoshenko theory.
- The electrical potential is a linear interpolation through the thickness.
- The piezoelectric material obeys a linear constitutive equation.

2.1. Reference system

The global and local Cartesian reference systems are mainly used as shown in Fig.1.

- Global reference system {O, x, y, z}.
- Local reference system {T, X′, Y′, Z′}.

A 3D beam is a solid of length L oriented in the longitudinal direction X'. The transverse area A with dimensions in $Y'Z'$ plane is orthogonal to X' and it is relatively small with respect to the longitudinal direction. Point c is the shear center of the beam. If the shear center c and the neutral point T do not coincide, a coupling between axial/bending and bending in x and y direction exists. The points T, g (neutral point, gravity center) and c coincide for homogenous solid sections.

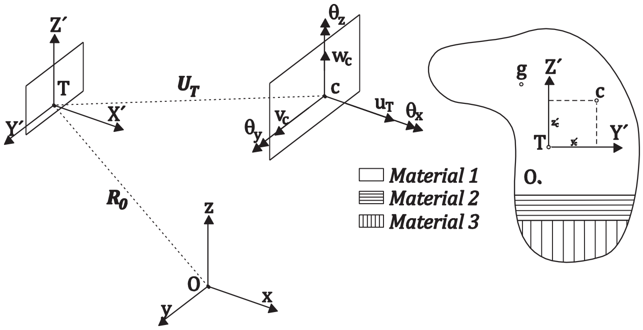

Fig. 1. Global reference systems: {O, x, y, z} and local reference systems {T, X′, Y′, Z′}. Gravity and shear centers (g and c).