Powder Technology 323 (2018) 149–154

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

DEM modeling on stress profile and behavior in granular matter

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ARTICLE INFO

Article history:
Received 12 July 2017
Received in revised form 7 September 2017
Accepted 3 October 2017
Available online 04 October 2017

Keywords:
Stress profile
Particle packing
Janssen model
Granular matter
DEM simulation

ABSTRACT

This paper presents a systematic numerical investigation of static stress profile within a confined granular packing using discrete element method. The vertical and horizontal stress profiles are carefully identified within the packing structure, and the effects of container diameter and friction coefficient on the stress distributions are systematically investigated and analyzed. The results show a quantitative agreement between the vertical stress profile gauged at the bottom and the stress profile calculated within the packing structure. I.e. beyond a hydrostatic region, the static vertical stresses saturate exponentially with the packing depth. A positive correlation between the saturation stress and the container size and a negative correlation with the friction coefficient can be observed. Besides, it is also found that the well-known Janssen coefficient (or Janssen constant) is not a constant value; it decreases asymptotically to a constant with the packing depth, which implies a non-constant friction effects existed between the granular assemblies and the boundaries. And the analysis of contact forces within the packing structure also indicates a stabilized condition beyond the hydrostatic region. It is believed that these findings could develop a comprehensive insight on the stress distribution of granular matter.

1. Introduction

Mechanical characteristic of static granular assemblies is still an open problem and has been of great interests in both engineering and physics communities [1,2]. One practical question lies in the prediction of static stress in a dry, non-cohesive granular particle assembly and has become the focus of a renewed attention [3–8], within which the stress distributions of granular matter in confined boundary conditions have excited doubts in some quarters [5,6] since the pioneering work conducted by Janssen in silo cells [9]. The relation between the vertical stress $\sigma_z(z)$ at the bottom and the depth of the packing bed was established with the assumption that the particle assembly could be treated as a continuous phase. Here the vertical stress is supposed to be deflected into horizontal stress $\sigma_h(z)$ by a phenomenological law, $\sigma_h(z) = k\sigma_z(z)$, where k is the Janssen coefficient (Janssen constant). The vertical stress or apparent mass of the granular matter measured at the bottom of the container does not increase linearly with the packing depth or filling mass, yet saturates exponentially as given by $\sigma_z(z) = \frac{\rho\phi g D}{4\mu k}[1 - \exp(-\frac{4\mu k z}{D})]$, where ρ and Φ represent the theoretical density of the particles and the overall fractional packing density of the packing bed respectively; g and μ represent the gravitational acceleration and the friction coefficient; z is the vertical depth of the packing bed and $D/4$ is the hydraulic radius. A great number of investigations were then conducted under different conditions both experimentally [10–20] and numerically [21–26] to test the validation of the Janssen model. For example, friction induced effects and boundary mobilization disturbance effects on the stress profiles were investigated [20,25,26]. The influences of particle size ratio, granular shapes as well as packing history were also partly explored by some researchers [10,11,13,14,19]. Previous work to some extent presents similar features of stress profiles, however, some studies based on physical experiments beg a particular question upon the effectiveness of Janssen's model [10], which has subsequently been numerically identified from the oriented stress linearity (OSL) model [21]. A modified model which can accurately capture the stress distribution in a confined packing (i.e. a hydrostatic region initially and an exponentially increasing zone with the packing depth) was proposed [10], and the effectiveness of this modified model was demonstrated by some studies [12,22,23]. However, to date, systematic studies on the Janssen model are still lacking. For instance, whether the stress measured at the container bottom reported from previous physical experiments could effectively predict the stress profile within the packing? And the influences of parameters such as container size and the friction coefficient which included in the Janssen equation still need to be further tested and studied. In addition, large differences of Janssen coefficient k can be seen from different researchers [9,10,13,14,16,18,22,24], so can the fitting value of k really interpret the physical meaning assumed by Janssen? Is this Janssen coefficient really a constant or dependent on the packing depth?

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https://doi.org/10.1016/j.powtec.2017.10.006
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All these concerns are important for a deeper understanding of stress profile and behavior of granular matter, and systematic researches in this regard are lacking, which need to be carefully investigated.

In this paper, systematic numerical study of the Janssen silo problem was carried out by using DEM (discrete element method) simulation. The vertical stresses were calculated both at the container bottom while charging and within the packing structure at different depths after charging. And the vertical stresses measured at the bottom were used to compare with the previous experimental and numerical results and to confirm whether the stresses measured at the bottom can effectively predict the stress profile within the packing structure. In addition, influences of various factors such as container diameter and friction coefficient on the stress distribution were carefully studied and the variation of contact forces in the packing structure with the packing depth was also discussed. We believe all these findings will help comprehensively understanding the behavior of confined particle packings and will be beneficial for improving the performance of granular matter and optimizing industrial processes in related areas.

2. Simulation method and conditions

In this paper, discrete element method (DEM) [27,28], known as an effective numerical method to simulate particulate systems [29,30], is used to simulate the packing behavior of granular matter. In DEM model, the translational motion and rotational motion for each particle (e.g. particle i) during packing are governed by Newton's second law of motion:

$$m_i \frac{d\boldsymbol{v}_i}{dt} = \sum_{j=1}^{p} \left(\boldsymbol{F}_{ij,n} + \boldsymbol{F}_{ij,s} \right) + m_i \mathbf{g} \tag{1}$$

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_{j}^{p} \left(\boldsymbol{T}_{ij,s} + \boldsymbol{T}_{ij,r} \right) \tag{2}$$

where m_i, $\boldsymbol{v}_i$, $\boldsymbol{\omega}_i$ and I_i are the mass, translational velocity, angular velocity, and inertia of particle i; p is the number of contacts with the considered particle i. The forces involved in the interaction between particles i and j include normal force $\boldsymbol{F}_{ij,n}$, tangential force $\boldsymbol{F}_{ij,s}$, and gravity; the torque acting on particle i from particle j consists of two parts: $\boldsymbol{T}_{ij,s}$ and $\boldsymbol{T}_{ij,r}$, which are generated by tangential force and rolling friction between these two particles, respectively. The details of the calculation of the forces and torques can be found from references [31,32].

To validate the simulation results, periodic boundary conditions are firstly applied in the horizontal direction within a cubic container, and a hydrostatic stress profile can be used to compare with the vertical stress calculated at the bottom and the within the packing structure. Then a series of simulations with cylindrical container wall are carried out. Meanwhile, in order to reproduce a similar charging process as that carried out in physical experiments, batch-wised feeding process is conducted in the simulation. The vertical stresses from the bottom after each intermittent feeding and within the final packing structure after feeding are respectively calculated. To obtain a comprehensive and general description, the simulations are carried out in five different sized cylindrical containers with each simulation running four times. Table 1 lists the parameters and corresponding values used in present simulation.

Table 1
Variables and corresponding values used in the simulation.

Name of variable	Symbol	Basic value	Variable range
Number of particles	N	45,000	5000–80,000
Particle density	ρ	2500 kg/m^3	–
Particle size	d	2 mm	–
Young's modulus	E	1.E7 N/m^2	–
Poisson's ratio	v	0.29	–
Sliding friction coefficient	μ	0.3	0.25–0.5
Rolling friction coefficient	μ_r	0.003	–
Damping coefficient	r_n	2.E-5	–
Time step	t	1.69E-5 s	–
Container diameter	D	20d	8–24

Normally, for non-cohesive particles, the static stresses are frequently studied and calculated by [33,34]:

$$\sigma_{\alpha\beta} = V^{-1} \sum_{n=1}^{p} \boldsymbol{l}_{\alpha} \boldsymbol{F}_{\beta,c} \quad (\alpha, \beta = x, y, z) \tag{3}$$

where $\boldsymbol{F}_{\beta,c}$ is the contact force vector and $\boldsymbol{l}_{\alpha}$ is the branch vector running from each particle center to its contact point. All these contacts are averaged within a considered representative volume V which is set to be a large bin with the radius equal to the container radius and with the height equal to 3d in vertical direction. It is worth noting that we also varied the bin height from 1d to 3d in the calculation, the results show that the 3d case can provide a clear and explicit stress profile with minor fluctuation and reasonable amount of data compared with the other cases. The vertical stress calculated at the bottom is given by:

$$\sigma_z = S^{-1} \sum \boldsymbol{F}_{z,c} \tag{4}$$

where S is the bottom area of the container and $\boldsymbol{F}_{z,c}$ represents the contact force between each individual particle and the container bottom in z direction.

3. Results and discussion

3.1. Vertical stress profile in the packing under periodic boundary conditions

Side wall effects can be avoided when the periodic boundary conditions are applied in horizontal directions, with which the vertical stress can be quantitatively described as:

$$\sigma(z) = \rho \phi g z \tag{5}$$

Fig. 1 shows the static vertical stress profile without wall effects. It can be seen that the vertical stress increases linearly with the packing depth, and each vertical stress value measured either at the bottom or within the packing bed could match the hydrostatic stress profile, which implies a comparable agreement between vertical stresses gauged at the bottom and within the packing bed. And this hydrostatic

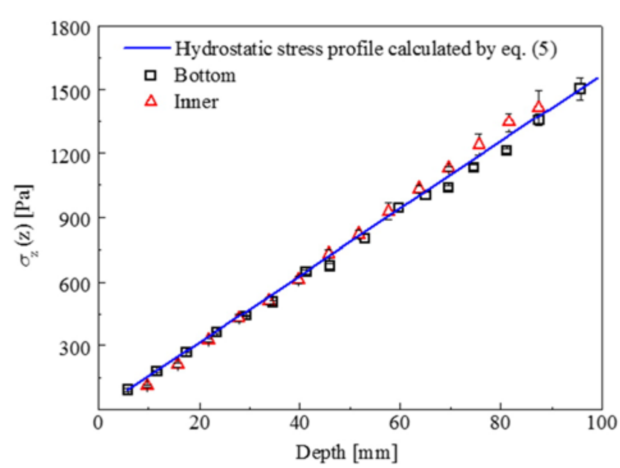

Fig. 1. Variation of static vertical stress with the depth of packing structure for the packing under periodic boundary conditions, the scattered symbols respectively represent the vertical stresses calculated at the bottom of the container and within the packing bed, and the error bar represents the fluctuation values obtained from four repeated simulations under the same conditions.