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Modelling of pastes as viscous soils – Lubricated squeeze flow

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ABSTRACT

Highly filled suspensions (or pastes) present complex rheological behaviour and squeeze flow testing is used frequently for rheological characterisation. The extent to which liquid phase migration (LPM) occurs in such tests, and the influence of material extruded from between the plates, was investigated in experiments supported by detailed modelling based on soil mechanics approaches. Lubricated squeeze flow (LSF) tests were conducted on a model saturated ballotini paste prepared with a viscous Newtonian binder, at plate speeds spanning two decades. The tests were simulated using a two-dimensional (2-D) axisymmetric finite element model with adaptive remeshing to circumvent mesh distortion. The paste was modelled as a viscoplastic soil (Drucker-Prager) to capture both rate-dependent effects at high shear rates and LPM at low shear rates. Capillary pressure was applied at the evolving free surface and the plate surfaces were modelled as frictionless for simplicity. Reasonable agreement was obtained between the measured and predicted squeezing pressure profiles at the highest solids volume fraction tested ($\phi_s = 60\%$). Agreement was poor at the lowest ϕ_s (52.5%), which was due to this paste formulation behaving as a suspension/slurry without a distinct yield stress. For the first time, the predicted squeezing pressure was resolved into components using an energy analysis. The squeezing pressure was dominated by the work required to deform the paste in the gap. This result is specific to highly viscoplastic pastes and persisted to small plate separations when most of the sample lay outside the plates. Characterisation of the yield stress from the 'shoulder' in the squeezing pressure profile was reasonably accurate at $h/h_0 \geq 96\%$ (9% estimated error). LPM was neither observed nor predicted at the plate speeds tested, despite the favourable pore pressure driving force, due to the high binder viscosity and the zero dilation angle in the simulations. The flow field was characterised using a novel flow mode parameter derived from the shear rate tensor. The paste was predicted to undergo pure biaxial extension between the smooth plates, and for the first time was predicted to undergo pure uniaxial extension external to the plates and (briefly) pure shear at the boundary.

1. Introduction

Particulate pastes are used widely to manufacture products such as agrochemicals, pharmaceuticals, and ceramic parts using techniques including ram extrusion, screw extrusion and injection moulding [1]. Pastes may be processed into the desired shape by forcing the material to flow through a gap (a die) of defined cross sectional profile, or in more complex cases by the filling of a mould. Pastes cease to flow upon removal of the driving force and retain their new shape: they are an example of a *soft solid*. Highly filled pastes demonstrate complex yield stress behaviour and hardening [2,3], wall slip [4] and migration of the liquid phase relative to the solids [5]. Some pastes are viscoplastic, reflecting the use of a highly viscous binder or (less frequently) a rate-dependent particulate matrix [6]. Others, such as mortar pastes, reflect 'frictional' rheology more reminiscent of dry particulate assemblies [7]. Some display transitional rheology that is affected by both friction at the interparticle contacts and viscous shear in the liquid binder. These are the focus of the current work.

Paste extrusion processes are challenging to model and simple solid or fluid mechanics-based rheological models do not explain experimental trends over all timescales, or for instance when the process geometry is altered. However, direct numerical simulation approaches such as the Discrete Element Method that model each particle individually would require what are currently prohibitive computer resources to simulate an extrusion process of realistic (industrial) size, as these can easily feature hundreds of billions of particles [8,9]. A practical middle ground is to study paste flow by combining a simple material testing protocol, such as a laboratory scale ram extrusion test or uniaxial compression test (squeeze flow or upsetting), with a low order finite element model [10]. This approach permits partial decoupling of the various

Abbreviations: CK, Carman-Kozeny permeability-porosity model; DP, Drucker-Prager constitutive model; DYS/SYS, dynamic/static yield surface; FEM, finite element modelling; FMP, flow mode parameter; LPM, liquid phase migration; LSF, lubricated squeeze flow; MRV, mesh refinement variable.

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Nomenclature

Roman

1/2/3-D	one/two/three-dimensional
$2D_{ij}$	deviatoric plastic strain rate tensor or plastic shear rate tensor, s^{-1}
Bn	generalised Bingham number for viscoplastic materials, –
BP_i	portion of total deviatoric power transferred to sample by upper plate that acts on the paste between the plates; external material is not included, W
CAX4P	ABAQUS element type used in all simulations
CK	Carman-Kozeny ϕ_s-permeability model
$CVPP_i$	combined viscoplastic and plastic power transferred to sample by the upper plate, W
d	volume-surface (Sauter) particle diameter, m
d_p	diameter of cylindrical pore, m
DP	Drucker-Prager constitutive model for yield of particulate solids
DYS	dynamic yield surface for a viscoplastic material
e	voids ratio, –
F	capillary suction force, N
FEM	finite element modelling
FMP	flow mode parameter, –
F_{sq}	squeezing force, N
g	acceleration due to gravity, m s^{-2}
G	shear modulus, Pa
h	current sample height, m
h_0	initial sample height, m
$III_{\dot{\gamma}}$	third invariant of the shear rate tensor, s^{-1}
$k_{1,2}$	constants in rheological model for pastes developed by Mascia and Wilson [3], Pa
k_t	tortuosity, –
K	permeability (hydraulic conductivity) of the solids skeleton, m s^{-1}
LPM	liquid phase migration
LSF	lubricated squeeze flow
MRV	mesh refinement variable used to guide the adaptive remeshing procedure
n	flow index during uniaxial compression for the pastes tested here, –
N_e	number of elements in the finite element mesh, –
p	hydrostatic effective pressure, Pa
p_0	reference effective pressure stress, Pa
$p_{e,q}$	local quasi-elastic power per unit volume, W m^{-3}
P	pore liquid pressure, Pa
P_0	initial value of pore liquid pressure in LSF simulation, Pa
P_c	capillary pressure at the free surface of the paste sample, Pa
P_{sq}	predicted average squeezing pressure at upper plate, Pa
P_{sqE}	measured average squeezing pressure at upper plate, Pa
P_{sqL}	predicted mean pore pressure at upper plate, Pa
QP_i	quasi-elastic power transferred to sample by upper plate, W
r	radial coordinate, m
R_f	fillet radius employed at outer edge of upper plate, m
R_p	radius of squeeze flow plates, m
s_{ij}	deviatoric stress tensor, Pa
s_t	surface tension of pore liquid, N m^{-1}
S	wetted surface area per unit volume of the bed, or wetted perimeter per unit area at the free surface, m^{-1}
SYS	static yield surface for a viscoplastic material
t	time, s
t_1, t_2	time at the beginning and end of a quasi-elastic stress increase, s
TDP_i	total deviatoric power transferred to sample by upper plate, W
u_r	radial velocity of paste (of solids skeleton when LPM is significant), m s^{-1}
u_z	axial velocity of paste (of solids skeleton when LPM is significant), m s^{-1}
U	liquid superficial velocity vector, m s^{-1}
v	velocity vector within solids skeleton, m s^{-1}
V	downward speed of upper plate, m s^{-1}
V_e	volume of an element of paste, m^3
VP_i	viscoplastic power transferred to sample by upper plate, W
x	orthogonal three-dimensional coordinate system, m
z	axial coordinate, m

Greek

$\lvert\dot{\gamma}\rvert$	square root of second invariant of the shear rate tensor, s^{-1}
$\dot{\gamma}_{ij}$	engineering deviatoric strain rate tensor, s^{-1}
$\lvert\gamma_{ij}\rvert_y$	engineering deviatoric strain required to reach yield from rest, —
$\dot{\gamma}_{p1-3}$	major, intermediate, and minor principal engineering deviatoric strain rates, respectively, s^{-1}
γ^e	engineering elastic deviatoric strain, –
γ^p	engineering plastic deviatoric strain, –
$\lvert\dot{\gamma}^{el}\rvert$	square root of second invariant of the elastic shear rate tensor, s^{-1}
$\dot{\gamma}^p$	plastic shear rate tensor, s^{-1}
$\lvert\dot{\gamma}^p\rvert$	square root of second invariant of the plastic shear rate tensor, s^{-1}
δe	increment in voids ratio, –
δ_{ij}	identity tensor (Kronecker delta), –
$\delta\gamma^p$	engineering plastic deviatoric strain increment, –
$\delta\varepsilon^p$	plastic strain increment, –
$\delta\varepsilon_v^p$	plastic volumetric strain increment, –
ΔP_{sqL}	increase in pore pressure relative to initial value in LSF simulation, Pa
Δt	duration of a time step in the LSF simulation, s
$\Delta\gamma_{ij,el}$	increment in engineering elastic deviatoric strain, —
$\dot{\varepsilon}$	strain rate tensor, s^{-1}
$\dot{\varepsilon}_p$	plastic strain rate tensor, s^{-1}
$\dot{\varepsilon}_{rr}$	radial strain rate within paste (solids skeleton when LPM is significant), s^{-1}
ε_v	total volumetric strain within the solids skeleton, –
ε_{zz}	logarithmic axial strain in sample, –
$\dot{\varepsilon}_{zz}$	radial strain rate within paste (solids skeleton when LPM is significant), s^{-1}
$\dot{\varepsilon}_{\theta\theta}$	circumferential strain rate within paste (solids skeleton when LPM is significant, s^{-1}
θ	circumferential coordinate, °
θ_c	contact angle between solid ballotini material, pore liquid, and air, °
κ	logarithmic elastic bulk modulus, –
μ	viscosity of liquid binder, Pa s
ξ_d	dilation angle for solids skeleton, °
ρ_l	density of pore liquid, kg m^{-3}
$\overline{\sigma}$	von Mises stress, Pa
$\overline{\sigma}_{DP}$	portion of $\overline{\sigma}$ arising from standard (rate-independent) DP model, Pa
σ_{ij}	local three-dimensional effective stress tensor, Pa
σ_{rr}	radial effective normal stress, Pa
σ_{rz}, τ_{rz}	both denote the radial-axial effective shear stress, Pa