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# Sensitivity analysis of Austin's scale-up model for tumbling ball mills – Part 3. A global study using the Monte Carlo paradigm

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ABSTRACT

The scale-up of batch grinding data is an attractive option for the modelling of industrial milling circuits. However, little is known in terms of the expected uncertainty attached to the predictions. This paper attempts to estimate this.

For simulation purposes, coarse and fine feeds were considered in the context of an open ball milling circuit and for three predefined feed flow-rates. Realistic fluctuations were allocated to the 25 input parameters making up the simulation model. The Monte Carlo approach was then used to simultaneously process the randomly generated input parameter values. Finally, average values and standard deviations on the 50% and 80% passing sizes of the mill product were calculated in order to obtain estimates of prediction uncertainties.

Global errors on the predicted full-scale mill product size were found to be as high as 70%. However, with appropriate assumptions on the scale-up correction factors, they were reduced to approximately 40%. It was also demonstrated that well planned batch tests and accurately determined batch data can further drop the error to as low as 20%. Finally, the scale-up model was found to work well when finer feeds were considered for milling at high flow-rates.



## 1. Introduction

This work is part of a sequel of papers that looked at the scale-up procedure for batch grinding data by Austin et al. [1]. The attractive feature of this scale-up model is that key input parameters are measured by means of batch grinding tests. No specific requirements are imposed on the dimensions of the laboratory mill or on the testing conditions. However, studies around the prediction accuracy associated with the scale-up procedure have been lacking. That is why, in the first paper, Mulenga [2] investigated the effects that variations induced on batch grinding parameters have on the scaled-up mill product. In the second paper, the effects of full-scale milling parameters were investigated [3]. The common drawback of these two studies was that the effects of input parameters were considered individually while keeping the other parameters constant. This hardly ever happen in an industrial setup; instead, fluctuations are present on several parameters at the same time. And in a peculiar instance, Mulenga [3] noted that the density of slurry feed had no effect on the mill product.

In this final follow-up paper, an attempt to address the aforementioned shortcomings is made with a two-fold goal: the first is to obtain a fair and realistic estimate of the global error to be expected when Austin's scale-up model is used and the second is to find alternative models that can capture better the effects of slurry concentration.

In order to meet the objectives set out, parameters relating to batch and full-scale milling were assigned acceptable levels of variation. Then, 1000 values within predefined ranges were generated for each milling parameter using a computer-based algorithm. The pseudo-random number values were simultaneously inputted into Austin's scale-up model applied to a ball mill operated in open circuit. After that, the average values and standard deviations recorded on the 50% and 80% passing sizes of the 1000 mill product were calculated. These were used to get an estimate of the global relative error on the mean inherent to the scale-up model. Finally, Austin's model was categorised in line with the level of uncertainty applicable to engineering projects.

It should be noted that the developmental cycle of engineering projects goes through three stages with their respective level of accuracy: a conceptual or scoping stage, a prefeasibility stage, and a feasibility stage. In this work, the intention was to see whether Austin's model incurs less than the ±25% maximum deviation required of engineering projects at a pre-feasibility stage [4].

## 2. Austin's scale-up model for batch grinding data

An overview of the scale-up procedure proposed by Austin et al. [1] is done in this section.

Austin's scale-up procedure starts with the preparation of mono-sized feed samples. These undergo a series of grinding tests in a laboratory mill run in batch mode following a technique known as one-size-

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fraction method [1]. The selection function and breakage function parameters characteristic of the material are measured as a result of this. These parameters can now be used in the scale-up model to predict the performance of any full-scale mill processing the same material.

The scaled-up the selection function is calculated as follows [1]:

$$S_i(d) = a_T\,(x_i)^{\alpha}\,\frac{1}{1+\left(\frac{x_i}{C_1\mu_T}\right)^{\Lambda}}\,C_2\,C_3\,C_4\,C_5 \tag{1}$$

where

$$C_1 = \left(\frac{D}{D_T}\right)^{N_2}\left(\frac{d}{d_T}\right)^{N_3} \tag{2}$$

$$C_2 = \left(\frac{d_T}{d}\right)^{N_0} \tag{3}$$

$$C_3 = \begin{cases} \left(\frac{D}{D_T}\right)^{N_1} & ,D\le 3.81\ m \\ \left(\frac{3.81}{D_T}\right)^{N_1}\left(\frac{D}{3.81}\right)^{N_1-N_4} & ,D>3.81\ m \end{cases} \tag{4}$$

$$C_4 = \left(\frac{1+6.6J_T^{2.3}}{1+6.6J^{2.3}}\right)\ \exp[-c(U-U_T)] \tag{5}$$

$$C_5 = \left(\frac{\varphi_c-0.1}{\varphi_{cT}-0.1}\right)\left(\frac{1+\ \exp[15.7\times(\varphi_{cT}-0.94)]}{1+\ \exp[15.7\times(\varphi_c-0.94)]}\right) \tag{6}$$

with $a_T$, $\mu_T$, $\alpha$, and $\Lambda$ being the laboratory-based selection function parameters whereas $N_0$, $N_1$, $N_2$, $N_3$, $N_4$, and $c$ are the scale-up correction factors.

The breakage function, on the other hand, is expressed as follows [2]:

$$B_{i,j} = \Phi_0\left(\frac{x_j}{x_0}\right)^{-\delta}\left(\frac{x_{i-1}}{x_j}\right)^{\gamma} + \left[1-\Phi_0\left(\frac{x_j}{x_0}\right)^{-\delta}\right]\left(\frac{x_{i-1}}{x_j}\right)^{\beta} \tag{7}$$

where $\beta$, $\gamma$, $\delta$, and $\Phi_0$ are collectively termed breakage function parameters. The term $x_0$ represents the standard particle size which has the value $x_0 = 1$ mm.

The following full-scale milling conditions are needed in Eqs. (1)–(7): mill diameter $D$, ball diameter $d$, slurry filling $U$, and mill speed $\phi_c$. Similarly, $D_T$, $d_T$, $U_T$, and $\phi_{cT}$ represent the batch grinding conditions used for laboratory testing.

## 3. Model of ball mill in open circuit

This section briefly describes the mathematical model of a full-scale ball mill operated in an open circuit configuration. The classical model formed the basis of the simulator built in this work and used to meet the objectives set out.

Given a cumulative feed size distribution that follows the Rosin-Rammler distribution function [5]:

$$\Psi_i = \Psi(x_i) = 1-\ \exp\left[-\left(\frac{x_i}{x_{63.2}}\right)^{\lambda}\right] \tag{8}$$

In Eq. (8), $\Psi_i = \Psi(x_i)$ is the mass fraction of feed passing size $x_i$; $x_{63.2}$ is the size at which the Rosin-Rammler distribution function $\Psi(x_i)$ has the value 0.632 while $\lambda$ is the characteristic slope of the feed size distribution $\Psi(x_i)$ when plotted on a Rosin-Rammler coordinate system.

When considering the full-scale mill, the mass fraction distribution of the feed corresponding to the cumulative distribution in Eq. (8) is given by: $f_i = \Psi(x_{i-1}) - \Psi(x_i)$. This feed size distribution, now termed $f_j$, is related to the mass fraction distribution of the product, $p_i$, discharged by the full-scale mill operated in open circuit as follows [3]:

$$p_i = w_i(t) = \sum_{j=1}^{i} d_{i,j}(t)\,f_j \tag{9}$$
$$\text{with } n\ge i\ge j\ge 1$$

The term $d_{i,j}(t)$ is a transformation matrix describing the breakage of the feed fraction of size $x_j$ into the size class $x_i$ of the discharged product. It is defined by the set of equations below:

$$d_{i,j} = \begin{cases} 0, & i<j \\ e_j, & i=j \\ \sum_{k=j}^{i-1} c_{i,k}\,c_{j,k}\,(e_k-e_i), & i>j \end{cases} \tag{10}$$

with

$$c_{i,j} = \begin{cases} -\sum_{k=i}^{j-1} c_{i,k}\,c_{j,k}, & i<j \\ 1, & i=j \\ \frac{1}{\overline{S_i}-\overline{S_j}}\sum_{k=j}^{i-1}\overline{S_k}\,b_{i,k}\,c_{k,j}, & i>j \end{cases} \tag{11}$$

and

$$e_j = \int_0^{+\infty}\exp\left(-\overline{S_j}\,t\right)\phi(t)\,dt \tag{12}$$

The average selection function $\overline{S_i}$ in Eqs. (11) and (12) is given by:

$$\overline{S}(x_i) = \overline{S_i} = \sum_k m_k\,S_{i,k} \tag{13}$$

where

$$m_k = \frac{d_k^{4-\Delta}-d_{k+1}^{4-\Delta}}{d_{\max}^{4-\Delta}-d_{\min}^{4-\Delta}} \tag{14}$$

with $m_k$ being the mass fraction of balls of diameter $d_k$ present inside the mill at steady state as a result of regular top-up of fresh balls of diameter $d_{\max}$; $d_{\min}$ is the minimum diameter of ball that is found inside the mill; $\Delta$ is a parameter dictating the wear rate model applied to grinding balls.

From the laboratory-based breakage function in Eq. (7) applied to a normalisable ore [1], it can be shown that the average breakage function $b_{i,j}$ in Eq. (11) is given by:

$$b_{i,j} = B_{i,j}-B_{i+1,j} \tag{15}$$

The next factor to consider in the model of the open milling circuit is the material transport. This is described by assuming that the full-scale mill consists of three perfectly mixed reactors in series: a large one of average residence time $\tau_2$ and two small ones of similar volume and average residence time $\tau_1$ [1,5]. The term $\phi(t)$ in Eq. (11) then be shown to reduces to

$$\phi(t) = \frac{\left[-\frac{t}{\tau_2}-\left(\frac{\tau_1}{\tau_1-\tau_2}\right)\right]\ \exp\left(-\frac{t}{\tau_2}\right)+\left(\frac{\tau_1}{\tau_1-\tau_2}\right)\ \exp\left(-\frac{t}{\tau_1}\right)}{\tau_1-\tau_2} \tag{16}$$