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Feasibility of measuring the pressure vs. volume relationship of compressible solids using a thick-walled cylinder

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ABSTRACT

According to the theory of thick-walled cylinder (TWC), the internal radial stress of the cylinder can be determined by measuring the hoop strain at the outer wall of the cylinder. We here investigate the feasibility of measuring the radial stress of compressible solids, such as a powder compact positioned inside of a cylinder, using the theory of TWC. Based on finite element analysis, we have shown that provided the geometry of the TWC is appropriately designed, TWC theory can be utilized up to significant volume strains of the specimen positioned in the TWC. The design guidelines for the geometry (thickness and internal radius) of the TWC are provided from the viewpoints of (1) the maximum magnitude of the volume strain up to which TWC theory is applicable, (2) the maximum measurable radial stress before yielding of the cylinder occurs, and (3) the magnitude of the hoop strain when the internal radial stress of the cylinder is very low, especially at the initial stage of powder compaction.

1. Introduction

Unlike bulk metals, compressible solids such as particulate materials (powder compacts), foams, rocks, and concretes demonstrate pressure-dependent yielding. Such solids can be yielded via the application of even, purely hydrostatic pressure. For compressible solids, the relationship between the pressure and volume strain is essential to the majority of constitutive models [1–23]. Therefore, there is a high demand to determine the pressure vs. volume strain relationship, especially for compressible solids [11,15,17,24–27].

The most extensively used method for measuring the pressure vs. volume strain relationship is conventional triaxial testing, especially for soils (a typical kind of particulate material) [17,25]. This method is generally limited to pressures of less than approximately 1 MPa, and triaxial instruments that allow measurements at pressures near 100 MPa are scarce due to their cost [22,23].

For the measurement of the pressure of compressible solids at pressures near 100 MPa (or even up to 300 MPa), the instrumented die [15,20,26,27] is one of feasible apparatus, inside which the compressible specimen is positioned and radial-stress-measuring sensors are embedded in the die itself. Then, an axial load is applied to the top of the specimen via a punch. After measuring the axial stress (σ_a, measured using the load cell) and the radial stress (σ_r, measured using the embedded sensor), the pressure of the specimen (p) is determined using the relationship $p = (\sigma_a + 2\sigma_r)/3$. The volume strain of the specimen is determined from the stroke of the punch providing the axial load.

In the powder compaction test, the axial load and the displacement of the stroke are routinely determined. However, measurement of the radial stress is not a simple task, which requires a special device such as the instrumented die (also an expensive tool). If a mean to economically measure the radial stress of the compressible specimen is available, experimentally determined pressure vs. volume data at pressures near 100 MPa should be more abundant than currently available data [11,24].

As an alternative method for economically determining the radial stress of a specimen, we considered a thick-walled cylinder (TWC). According to the theory of thick-walled cylinders (open-ended condition), the internal radial stress (σ_r) of the cylinder can be simply determined using knowledge (measurement) of the hoop strain (ε_θ) at the outer wall of the cylinder [28]:

$$\sigma_r = \frac{b^2 - a^2}{2a^2} E\varepsilon_\theta, \tag{1}$$

where a and b are the inner and outer diameters of the cylinder, respectively, and E is the Young's modulus of the cylinder material. The theory of TWC considers situations where the radial stress in the inner wall (internal pressure) is applied throughout the whole height of the cylinder, causing the whole cylinder to expand radially.

However, the specimens located in the TWC during practical compression tests provide radial stress to only the inner wall portion corresponding to the height of the specimen being compressed. As a

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result, the radially un-stressed portion of the cylinder prohibits radial expansion of the cylinder portion that is in contact with the specimen. Therefore, the direct application of TWC theory (Eq. (1)) to a cylinder being radially stressed in only a limited region leads to erroneous determinations of the internal radial stress using the measured hoop strain. However, if the height of the specimen is not too much shorter than the height of the cylinder, i.e., within the limited stroke of the punch, there may be a possibility of measuring the radial stress of the specimen using TWC theory. Based on this reasoning, we here investigate whether measuring the radial stress of the specimen using the TWC is feasible. Through a systematic numerical analysis (finite element analysis), this study will show that, provided the geometry of the TWC is appropriately designed, TWC theory can be applied up to a significant volume strain of the compressible specimen (stroke of the punch). We will also provide design guidelines for the geometry of the TWC (thickness and internal radius) for measuring the radial stress of a specimen using a TWC.

2. Analysis method

The concept of the experimental setup using the TWC is schematically illustrated in Fig. 1(a), and (b) presents the geometry of the 2D-axisymmetric model for numerical analysis. In Fig. 1(b), H_o is the height of the cylinder, H is the current height of the specimen applying the internal pressure (radial stress, σ_r) to the cylinder, and D_i and t are the inner diameter and the thickness of the cylinder wall, respectively. In this study, the current height of the specimen is expressed in terms of a normalized term, H/H_o. The position (z) along the height direction of the cylinder is also normalized with respect to the height of the cylinder as z/H_o.

The geometries of the models considered in this study are summarized in Table 1. In experiment, the height of the specimen decreases with increase of the radial pressure. In order to depict such a phenomenon, we considered a number of models with varying H/H_o ratios at two different σ_r values, 1 and 40 MPa. In order to create design guidelines for the geometry (D_i and t) of the TWC, we also considered models with varying D_i and t. For each D_i, the H/H_o ratio was varied; specifically for each H/H_o ratio, we varied the wall thicknesses between $t = 1, 2, 5,$ and 10 mm.

A typical example of the finite element model for the TWC is presented in Fig. 2. The sensitivity of the mesh size was checked separately, and the mesh size shown in Fig. 2 was confirmed to coincide with the analytical solution (Eq. (1)). The biased mesh in the radial direction is helpful for obtaining the analytical solution (Eq. (1)) for a given number of meshes.

A stainless steel, AISI 304L, was used as the material for the TWC. An elastic-plastic material model with the Ludwik work-hardening law was employed to describe the mechanical behavior of the TWC. The Ludwik hardening law is given by

$$\sigma = A + B\varepsilon^n, \qquad (2)$$

where σ and ε are the equivalent stress and the equivalent plastic strain, respectively, and A, B, and n are the material parameters illustrated in Table 2 [29].

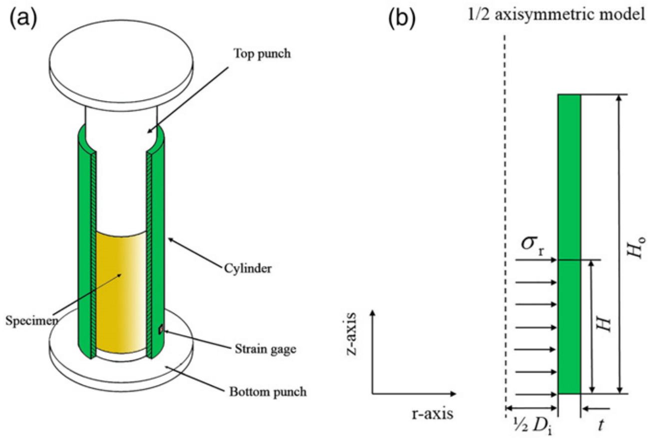

Fig. 1. (a) Conceptual diagram of the experimental setup using a TWC and (b) the geometry of the 2D-axisymmetric model for numerical analysis. H/H_o is the normalized current height of the specimen with respect to the height of the cylinder (H_o = 50 mm).

Table 1
Geometries of the cylinders considered in this study.

D_i (mm)	H/H_o (H_o = 50 mm)	t (mm)
10, 20	1, 0.75, 0.5, 0.25, 0.2, 0.1	1, 2, 5, 10

3. Results and discussions

We first determine the feasibility of measuring the radial stress of specimens using TWCs; this was accomplished through the use of finite element models with varying specimen heights, as presented in Section 3.1. Then, the design guidelines for the wall thickness (Section 3.2) and the inner diameter (Section 3.3) are determined by considering: (1) the maximum magnitude of the volume strain of the specimen up to which TWC theory is applicable, (2) the maximum measurable radial stress before cylinder yielding occurs, and (3) the magnitude of the hoop strain when the magnitude of the internal pressure (radial stress) of the cylinder is very small, especially at the initial stage of the compression testing.

3.1. Feasibility of measuring the radial stress

The profiles of the hoop strain at the outer surface of the wall along the direction of the cylinder height are presented in Fig. 3 for varying current heights (H/H_o) of the specimen, with a radial stress (internal pressure) of 40 MPa applied to the inner wall of the cylinder. The results in Fig. 3 were obtained for D_i and t values of 10 mm and 1 mm, respectively. In Fig. 3, the position along the direction of the cylinder height was normalized as z/H_o. As can be seen in Fig. 3, when the radial stress is applied across the whole cylinder height ($H/H_o = 1.0$), the hoop strain at the outer wall is uniform across the whole height of the cylinder ($0 < z < 100\%$) and the value of the hoop strain (910 micro-strain) is consistent with the results of TWC theory (analytical solution; Eq. (1)). This finding confirms the reliability of the numerical analysis carried out in this study with respect to the analytical solution.

Consider now the case when the current height (H/H_o) of the specimen is 0.5. In this case, hoop strain does not appear at the cylinder portion for z values higher than approximately 68%, i.e., this cylinder portion does not expand radially and actually constrains the radial expansion of the portion that is in contact with the specimen. However, at the position where z/H_o is less than approximately 33%, the measured value of the hoop strain at the outer surface of the wall is consistent with the analytical solution.

If we consider the case when the current height (H/H_o) of the specimen is 0.25, the measured hoop strain is consistent with the analytical solution for z/H_o values less than approximately 8%. This finding indicates that, provided the hoop strain is measured near the bottom of the cylinder where $z/H_o < 8\%$, the measured hoop strain is consistent with the analytical solution provided by TWC theory. In other words, the radial stress can be measured up to a significant magnitude of the volume strain[1] ($|\ln H/H_o| = |\ln 0.25| = 1.39$) using the analytical

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[1] In order to accurately calculate the volume strain, the radial displacement of the cylinder should be considered. However, the radial displacement is <0.002% when an internal pressure (radial stress) of 40 MPa is applied (D_i = 10 mm and t = 2 mm). For this reason, the effect of the radial displacement is neglected when we discuss the magnitude of the volume strain in this study.