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An impulse based model for spherical particle collisions with sliding and rolling

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ABSTRACT

Particle impact behavior description is significant for dense multiphase flow simulation and particle deposition prediction. The paper presents a novel impulse based particle collision model considering sliding and rolling. The collision model includes three components, normal, tangential and adhesion model. Normal collision model is established to predict the normal coefficient of restitution (COR), contact area, total normal impulse etc., by dividing the collision process into three compression stages (elastic, elastic-plastic and fully plastic stages), and an elastic recovery stage. The tangential and angular velocities are calculated based on the normal impulse and the determination of sliding or rolling contact. It is verified that the direction of tangential relative velocity remains the same during the whole collision process, with the assumption that the deformation of the particles does not affect the tangential forces. The adhesion loss is only activated in the recovery stage using Dunn's model, which describes the adhesion force as an idealized line force acting on the contact radius. Several cases are tested to verify the proposed model, including low velocity normal impact of particle and wall, oblique impact of particle and wall with or without spin, sphere moving on a flat plate under gravity, and particle-particle collision. The results show that the proposed model is able to predict the experimental measurements with good accuracy and captures the essential physics of particle-particle and particle-wall impacts.

1. Introduction

Granular and fluid-particulate flows are present in many natural and man-made systems in the chemical, pharmaceutical, power, and aerospace industries. However many research challenges remain in the realistic quantitative and predictive simulations of multi-particle systems [1,2], and in the characterization of particle-particle and particle-surface interactions [3,4]. The accuracy and effectiveness of the simulations are largely dependent on the model description of the impact behavior of particle-particle collisions and particle-surface collisions. In fact, particle collision is a very complex event which not only depends on the impact velocity but also on material properties such as Young's modulus, Poisson's ratio, yield and tensile strengths, surface energy, friction coefficients, etc.

Typically, collisions are characterized by the coefficient of restitution (COR), which is a measure of the energy lost during impact. Energy losses during collisions are described by losses attributed to plasticity, viscoelasticity, adhesion, friction, and other dissipative mechanisms. Plastic deformation is very prevalent during high velocity impacts between particles and is one of the factors contributing to irreversible losses. Friction forces are generated by contact pressure during impact and the relative motion of the two colliding bodies, and affect the resultant tangential velocities and rotation speed. Furthermore, adhesion forces play a dominant role in energy loss for micron-to-submicron-sized particles at low impact velocity [5].

The interaction of two particles or particle-surface is still a subject of research and no general solutions exist. Various impact models have been developed based on the principles of energetics and mechanics, such as spring-dashpot model [6], Jackson-Green (J-G) model [7], and Stronge model [8]. Yu and Tafti [9] developed a particle-surface elastoplastic impact model based on Stronge's model with a new elastic recovery model and also considering adhesion forces for particle-surface impacts. The model was verified with experimental data for millimeter sized as well as micron sized particles impacting a surface. This model was further extended to 3D particle-particle collisions [10]. These studies mostly focused on modeling normal collision dynamics, thus, rotational motion of the particles, sliding and rolling were not critically evaluated in the theoretical descriptions.

Mindlin [11] showed that when two spheres under a normal load are subjected to a tangential force, then sliding occurs if the tangential force is larger than the friction coefficient times the normal force, but at lower tangential forces, the particles remain stuck in the central circular contact region but undergo sliding in the surrounding annulus.

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Nomenclature

a	contact radius (m)
b_1 b_2	constant coefficients of yield stress
C_R	surface roughness factor
C_H	Hertz damping coefficient
E	Young's modulus (GPa)
E_*	effective Young's modulus of particle and target (GPa)
e	coefficient of restitution (COR)
e_n	normal COR of impact
e_t	tangential COR
F_n	normal contact force (N)
f_0	circumferential tension of the adhesion force per unit length (N)
I	moment of inertia (kg·m^2)
m	mass (kg)
m_*	effective mass of particle and target (kg)
$\mathbf{n}$	center vector
$\mathbf{n}_\omega$	angular velocity direction vector
P	impulse (N·s)
p	contact pressure (MPa)
R	radius of particle (m)
R_*	effective radius of particle and target (m)
$R_*^{\ r}$	effective radius during recovery stage (m)
$\mathbf{r}$	distance vector
$\mathbf{V}$	velocity vector (m/s)
$\mathbf{V}_{1O}$, $\mathbf{V}_{2O}$	center velocity vectors of particles (m/s)
W_A	work of adhesion forces (J)
W_{diss}	work of dissipative forces (J)

Subscripts

c	critical value between elastic and elastic-plastic stages
cp	critical value between elastic-plastic and full plastic stages
i	initial
r	recovery stage, resultant
t	tangential
n	normal

Superscripts

c	critical value between sliding and rolling
c'	critical value between rolling and constant velocities
e	effective value at elastic compression stage
e'	effective value at recovery stage
r	recovery stage

Greek alphabet

α	impact angle (degree)
γ	surface free energy (J/m^2)
$\triangle\mathbf{V}$	relative velocity vector (m/s)
$\triangle\boldsymbol{\omega}$	relative angular velocity vector (rad/s)
δ	deformation displacement (m)
$\delta_{\max}$	maximum displacement (m)
δ_r	recovery displacement (m)
μ_r	rolling friction coefficient
μ_s	sliding friction coefficient
ν	Poisson's ratio
$\boldsymbol{\omega}$	angular velocity vector (rad/s)
σ_Y	Yield stress (MPa)

Mindlin and Deresiewicz [12] further investigated varying normal and tangential forces and showed that the response of the system to small changes depends on the previous loading history. Maw et al. [13,14] developed a procedure to identify the status of motion in the tangential direction, using a suitable series of functions to describe the distribution of tangential displacement for elastic deformation and sliding. From their analysis they concluded that simple rigid body theory of impact in the tangential direction, which does not take tangential elastic deformation into consideration, gives a reasonable approximation at low and high angles of incidence, but is unsatisfactory at intermediate angles.

In the earlier work cited above sliding resistance together with elastic deformation was assumed to play the dominant role in tangential impact dynamics. However, in the context of granular assemblies, Oda et al. [15] showed through biaxial compression tests on assemblies of oval cross-section rods that particle rolling appeared to be a major microscopic deformation mechanism, especially for large inter-particle friction. Since then there have been many efforts to include rolling resistance in modeling particle collisions in free flight as well as in granular assemblies.

All of these developments have been in the framework of the Discrete Element Method (DEM) using the collision model framework proposed by Cundall and Strack [6] using equivalent spring-dashport (or soft-sphere model) to model collisions in the normal and tangential directions. Iwashita and Oda [16,17] proposed a modified discrete element method (DEM) in which an additional set of an elastic spring, a dashpot, a no-tension joint and a slider are installed to model the rotational motion together with the sliding to simulate shear band formation. Zhou et al. [18] introduced the rolling friction torque based on the experimental and theoretical analysis of Beer and Johnston [19] to simulate the formation of sand piles. Tordessillas and Walsh [20] incorporated rolling resistance and contact anisotropy in a non-local micropolar model to investigate the bulk deformation of granular media. Kondic [21] presented a soft sphere impact model including sliding and rolling. Zhu and Yu [22] studied the equilibrium, stability and pure rolling problems of a sphere moving on a flat plane. Luding [23] presented an overview on the use of the soft-sphere model with sliding and rolling for simulating small cohesive particles in the range 0.1–10 μm to predict their macroscopic behavior. Jiang et al. [24] proposed a three-dimensional contact model incorporating rolling and twisting resistances at inter-particle contact using the spring-dashpot method. Ai et al. [25] identified four different classes of rolling resistance models that are commonly used in DEM. They proposed a soft particle model and validated it to be more general and suitable for modeling problems involving both dynamic and pseudo-static regimes. Wang et al. [26] presented the detailed derivation method for the 3-D cases and validated the result by checking two special cases of rolling.

In real physical systems, energy loss due to rolling is from two main sources; mechanisms that contribute to hysteresis at the point of contact (micro-sliding, visco-elasticity, plasticity etc.) and the effects of shape [27]. Rolling friction coefficients are hard to confirm, and several definitions of rolling are proposed. Wensrich and Katterfeld [27] presented a method for estimating the coefficient of rolling friction for simplified shapes using a simple geometric argument based on the eccentricity of contact.

The measurement of the impact between particle and surface makes details of particle interactions easy to understand and provide validation for different theoretical impact models. While some experiments are focused on the normal coefficient of restitution [28], Kharaz et al. [29,30] measured the oblique impact behavior of spherical particles at low velocities. Kleis and Hussainova [31] investigated the particle-wall impact process using a Laser Doppler Anemometer (LDA) measuring technique. Dong and Moys [32] measured the properties of impacts between a 44.5 mm steel ball and a steel flat surface with and without initial spin. The progress of experimental methods for particle collisions provides an efficient avenue to validate theoretical models.

The current investigation develops an impulse based model for particle-particle and particle-surface impacts including translational and angular velocities, and inclusion of sliding and rolling regimes during impact. To the best of our knowledge, no previous study has used an impulse based treatment of sliding and rolling to model collisions. The model is based on a rigid body assumption in the tangential direction,