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Analytical description of solid particles kinematics due to a fluid flow and application to the depiction of characteristic kinematics in cold spraying

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ABSTRACT

In several multiphase flow applications such as fluidization, thermal spraying, atomization manufacturing and so on, the Newton's law is widely enacted to formulate the particle/fluid kinematic interaction and then to compute particles kinematics. This paper provides analytical solutions of the Newton's law in its time-dependent formulation or simplified formulation, the latter being a reduction of the time dependent problem into a spatial description of the particle motion. It was found that the velocity solution is strictly similar in both cases so that the simplified formulation is viable. The W_{-1} branch of the Lambert's function yields the analytical particle residence time and the particle velocity which enable to characterize particle kinematics and capabilities of cold spraying. Typical particles residence time is of about 10^{-7}–10^{-6} s and a typical characteristic duration is of about 10^{-4}–10^{-3} s to produce a deposit layer. This explains the high productivity rate of cold spraying among innovative additive manufacturing processes. Theoretical limitations of cold spraying are depicted using analytical solutions. Variances of particles velocity are mapped depending on both particle nature and Mach number used in cold spraying. According to analytical laws, the particle velocity using air propellant gas is limited to 600 m/s–1000 m/s for the situation of low particle density-diameter combination ($\rho_p$$*$$D_p$) experienced in cold spraying. Helium increases this kinematic limitation up to 1000 m/s–1600 m/s. Such analytical depictions characterize and facilitate a theoretical selection of process parameters including suitable particle features depending on gas nature and kinematic capabilities of cold spraying.

1. Introduction

Today, the additive manufacturing method offers new perspectives of technological solutions. Against conventional manufacturing processes based on subtractive machining, shaping or moulding, additive manufacturing allows a direct shaping that enables the concretization of complex design or novel components performance [1]. With the development of powders deposition method, there is substantial progress in terms of structural functionalization possibilities at various scales using standard materials. The cold gas dynamic spray (CGDS) process contributes for a significant part in such achievements since it has a convincing capability to produce several functional performances for various industrial applications [2,3]. The process transforms primary powders, which can consist of single powder nature or a combination of different powders, into a bulk component by a layer-by-layer continuous consolidation [4], these powders being sprayed onto a substrate under high velocity collision conditions using supersonic gas flow [5, 6]. Pressurized gas is fed into a De Laval nozzle through which this gas is expanded and then generates supersonic velocities that accelerate the primary powders injected into the nozzle. Since the particle velocity is a major parameter that decides the adhesion onto the substrate [7,8], its determination is crucial for the depiction of the deposition capability of a cold spray system. The particle in-flight behaviour has to be characterized depending on the process parameters including the gas inlet conditions, the powders characteristics, and the nozzle dimensions. This can be performed experimentally using laser velocimetry but the interdependence of the process parameters can easily involve tremendous task and costly tests so that conducting virtual testing instead becomes more affordable.

For a two-phase flow involving a solid particle/fluid interaction, the particle motion due to the action of the fluid is described by a position and a velocity at a time t in accordance with the Newton's law. In cold spraying literature, a simplification has been enacted to obtain the particle velocity along the nozzle. The time dependent problem is reduced into a spatial description of the particle motion:

$$\frac{dV_p}{dt} = V_p \frac{dV_p}{dx} \quad (1)$$

By this way, at any coordinate along the nozzle axis, the particle velocity change due to the drag force generated by the flowing gas is

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Nomenclature

Latin-script symbol

r	Nozzle radius along the nozzle axis, (m)
r^*	Radius of the nozzle throat, (m)
C_1, C_2	Dimensionless coefficient, (-)
C_D	Drag coefficient of particle, (-)
D	Diameter, (m)
F_D	Drag force, ($kg.m.s^{-2}$)
m	Mass, (kg)
M	Mach number, (-)
P	Pressure, (Bar)
Re	Reynolds number, (-)
Rs	Specific gas constant, ($J.kg^{-1}.K^{-1}$)
t	Time, (s)
T	Temperature, (K)
V	Velocity, ($m.s^{-1}$)
$V_{p\ cr}$	Critical particle velocity for adhesion, ($m.s^{-1}$)
x	Coordinate variable, (m)
X	Position, (m)
W	Lambert's function, (-)
W_{-1}	Branch of the Lambert's function, (-)

Greek-script symbol

γ	Ratio of specific heat, (-)
μ	Dynamic viscosity, ($kg.m^{-1}.s^{-1}$)
ρ	Density, ($kg.m^{-3}$)
τ	Intermediate variable, (m^{-1})

Subscript symbol

p	Particle
g	Gas
0	Reference denoting an initial condition
i	Reference denoting input conditions for the gas parameters

determined. Assuming that the particle velocity is small enough with respect to the gas velocity, its variation was found to be proportional to the gas velocity and the square root of the coordinate x [9]. The coordinate x is also the nozzle coordinate and the variation of x represents a distance travelled by the particle over the value of its diameter. For a wide application of this consideration, i.e. without any assumption between particle speed and gas speed, the distribution of V_p along the nozzle axis is generally computed via numerical integration method. Together with an isentropic flow formulation governing the gas flow through the nozzle, this approach has been widely used for a substantial understanding of the behaviour of the CGDS process [6, 9–11], the process parameters selection [12,13] or the process optimization [14–16]. Physically realistic results were found but the accuracy of this spatial formulation of the particle velocity variation along the nozzle coordinate has not been really discussed. This paper suggests a comparison of this formulation with the time dependent formulation using analytical solutions of the velocity variation described by the Newton's law. Typical residence times of the particles for a given travelled distance will be characterized as well as the variance of characteristic particles times, in cold spraying. Furthermore, a depiction of the CGDS capability using these analytical solutions will be performed.

2. Particle motion equations and exact solutions

The kinematic behaviour of the particles during cold spraying is a typical case of interactions between solid particles and moving fluid for which the governing equation was already found. Theoretical backgrounds on the description of the physical particle/fluid interactions suggest a general formulation denoted by "one-way coupling" wherein the fluid flow decides the particles motion. The capability of solid particles to move within a flowing fluid has been modelled in terms of external forces acting on the particles. The formulation relies on the consideration of spherical particles without particle/particle interactions. Assuming a low gas density with respect to the particle density, the drag force is a predominant external force. This situation prevails in cold spraying since the density ratio between gas and particle is in the range of 10^{-3}. Furthermore, the gravity effect can be neglected considering the circumstance of high speed flow during the cold spray process combined with the use of micron sized particles. The spherical particle is sufficiently small so that the drag force becomes the major source of motion, nearly uniform over the particle surface. These conditions can be enacted for cold spraying and the drag force F_D, in its general formulation, is expressed as follows:

$$F_D = \frac{3}{4}\frac{\rho_g}{\rho_p}\frac{m_p}{D_p}C_D(V_g - V_p)|V_g - V_p| \tag{2}$$

where ρ, m, d and V denotes respectively a density, a mass, a diameter and a velocity. The subscript p and g mean particle or gas parameter. C_D is the drag coefficient which can be defined on whether the particles experience a subsonic, sonic or supersonic regime. Correlations details are provided in the appendix part.

2.1. Time-dependent kinematics formulation

The Newton law describes the time-dependent Lagrangian formulation of the particles kinematics within the gas flow. The particle motion is defined by the particle velocity V_p at the particle position X_p and at a time t:

$$m_p \frac{dV_p}{dt} = F_D \tag{3}$$

In cold spraying, the gas velocity is generally higher than the particle velocity throughout the nozzle so that $|V_g - V_p| = (V_g - V_p)$, and the equations of motion along the nozzle axis are:

$$\frac{dX_p}{dt} = V_p \tag{4}$$

$$\frac{dV_p}{dt} = \frac{3}{4}\frac{\rho_g}{\rho_p}\frac{C_D}{D_p}(V_g - V_p)^2 \tag{5}$$

For a given ρ_p, ρ_g, d_p, V_g at a time t and considering the variation and correspondence defined in Eq. (6), the solution of $X_p(t)$ (Eq. (7)) and $V_p(t)$ (Eq. (8)) can be established as detailed in the appendix section.

$$\left.\begin{array}{r} time: t_0 \rightarrow t \\ V_p: V_{p0} \rightarrow V_p(t) \\ X_p: X_{p0} \rightarrow X_p(t) \end{array}\right\} \tag{6}$$

$$X_p(t) = X_{p0} + V_g(t - t_0) - \frac{4}{3}\frac{\rho_p}{\rho_g}\frac{D_p}{C_D}\, log\left[1 + \frac{3}{4}\frac{\rho_g}{\rho_p}\frac{C_D}{D_p}(t - t_0)(V_g - V_{p0})\right] \tag{7}$$

$$V_p(t) = V_g - \frac{1}{\frac{3}{4}\frac{\rho_g}{\rho_p}\frac{C_D}{D_p}(t - t_0) + \frac{1}{V_g - V_{p0}}} \tag{8}$$

The equation of the particle position Xp(t) can be inverted to give the particle residence time for a travelled distance $dX_p = X_p(t)\text{-}X_p(t_0)$.