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Analytical study on char combustion of spheroidal particles under forced convection

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ABSTRACT

Direct combustion of biomass fuel is an efficient means of renewable energy utilization. Biomass particles are generally non-spherical, which affects their combustion properties. The particles either in prolate or oblate spheroid shapes are involved in this paper. Using the ellipsoidal coordinates, a theoretical study is conducted on the char combustion of the spheroidal particle under forced convection. Expressions for the char combustion rates of both prolate and oblate spheroids are obtained in diffusion controlled and diffusion-kinetics controlled regimes. The char combustion rate for the spheroidal particle is influenced by the Sherwood number for gas-particle mass transfer. It is enhanced by the increase in the particle aspect ratio for both prolate and oblate spheroids with the same surface area.

1. Introduction

Biomass energy is a clean and inexhaustible energy resource. The thermochemical conversion of biomass fuel, such as direct combustion or co-firing with coal for heat and electricity production, has been found to be a promising approach to utilize biomass energy. During combustion processes, biomass particles undergo a series of physical and chemical changes. Apart from the processes of drying, devolatilization, ignition, and volatile combustion, the char combustion is an indispensable part of biomass particle combustion. The biomass char combustion is complicated by the particle elemental composition, particle structure, convective mass and heat transfer, reaction kinetics, and gas turbulence. It is also influenced by the particle physical properties [1,2]. The particle shape raises additional concerns. Biomass particles are generally irregular-shaped, exhibiting non-spherical characteristics with various aspect ratios (length/diameter). Naturally, particle shapes are largely dependent on the fuel preparation systems adopted. Besides near-spherical particles, there are generally other irregular-shaped biomass particles [3,4]. For example, straw and switchgrass particles can be approximately described as prolate spheroid or cylindrical particles, while wood chip particles tend to be more oblate or disk-like. Non-spherical characteristics enhance the external surface areas and then influence the combustion dynamics and heat and mass transfer properties of particles in comparison with spherical particles having the same volume [5].

The effect of particle shape on biomass char combustion is a major concern in developing a mathematical model for the burning of biomass particles [5–8]. A number of theoretical investigations have been carried out to develop the char combustion model and to explore the heat and mass transfer characteristics for a spheroidal particle. Grow [6] proposed an integral expression for the oxygen mass flux at the surface of a prolate spheroid under quiescent environment. Based on Grow's work [6], Gera et al. [5,7] defined an enhancement factor that accounts for the effect of non-spherical particle on the char combustion rate. The factor was calculated by solving the ratio of the average oxygen mass flux at the particle surface to that for the spherical particle having the same surface area. The burning enhancement factor for the spheroidal particle was further developed by Yin et al. [8]. A simple algebraic enhancement factor was proposed to replace the complex integral expression.

Although the char combustion models for spheroidal particles have been developed, they still have some problems: (1) The resultant char combustion rate contains an integral term, which is not easy for engineering application. (2) The theoretical expression is restricted to the diffusion controlled regime of char combustion under quiescent environment. To further develop the char combustion model for spheroidal particles, the fully analytical solutions for the char burning rates of both prolate and oblate spheroids were obtained in the diffusion controlled regime [9]. The char combustion of prolate spheroidal particles was studied in the diffusion-kinetics controlled regime, but the forced convection was considered with the aid of burning enhancement factor [10]. On the basis of previous studies, the char combustion rates for both prolate and oblate spheroidal particles are investigated in the

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Nomenclature

a, b, c	semi-axes of ellipsoid (m)
A	particle surface area (m^2)
B_h	pre-exponential factor (kg/($m^2 \cdot s \cdot Pa$))
D	diffusion coefficient (m^2/s)
E	scale factor (m^3)
E_h	activation energy (J/mol)
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	Cartesian coordinate unit vectors (m)
g	mass flux (kg/($m^2 \cdot s$))
G	mass flow rate (kg/s)
H	scale factor (m^{-1})
k	mass transfer coefficient (m/s)
m_k	particle mass (kg)
m_{k0}	particle initial mass (kg)
M	particle aspect ratio
M_s	gas molecular weight at particle surface (kg/mol)
M_{ox}	oxygen molecular weight (kg/mol)
$\mathbf{n}$	unit vector outward normal to concentric ellipsoid surface (m)
p_s	gas pressure at particle surface (Pa)
r	sphere radius (m)
R	universal gas constant (J/(mol·K))
Re_k	particle Reynolds number
Sc	Schmidt number
Sh	Sherwood number
Sh_0	Sherwood number under static conditions
T_k	particle temperature (K)
x, y, z	Cartesian coordinates (m)
Y_{ox}	oxygen mass fraction
Greek symbols	
β	mass stoichiometric ratio for char combustion
ξ, η, ζ	ellipsoidal coordinates (m^2)
Π_s	ellipsoid surface
ρ	gas density (kg/m^3)
ρ_p	particle material density (kg/m^3)
Subscripts	
c	char
os	oblate spheroid
ox	oxygen
ps	prolate spheroid
s	particle surface
sp	sphere
sup	film boundary
∞	surrounding environment

present paper. The analytical expressions obtained in [9] are extended to consider the influences of convective mass transfer between gas and particle. They are also extended to the diffusion-kinetics controlled regime of char combustion.

2. Char combustion model for spheroidal particle

2.1. Total mass flow rate at the particle surface

A single char oxidation reaction is assumed to occur on the spheroidal particle surface. No spatial reaction takes place in the gas layer near the particle. Consider the Stefan flow at the spheroidal particle surface. There is no relative motion between the particle and the gas environment. The flow near the particle is symmetric. The total oxygen mass flux at the surface of a concentric spheroid is composed of two parts: the diffusional flux of oxygen and the oxygen mass flux associated with the bulk flow. It is written as [9]

$$g_{ox} = Y_{ox} g - \rho D \frac{\partial Y_{ox}}{\partial n} \tag{1}$$

where Y_{ox} is the oxygen mass fraction, g and g_{ox} are the total and oxygen mass flux, respectively, and n represents the outward normal direction at the surface of the concentric spheroid.

It is convenient to use the ellipsoidal coordinates in the subsequent derivation. The surface of an ellipsoid in the Cartesian coordinates satisfies the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. The three fixed positive numbers a, b, and c describe the ellipsoidal shape. The ellipsoidal coordinates (ξ, η, ζ) are the roots of the equation $x^2/(a^2 + s) + y^2/(b^2 + s) + z^2/(c^2 + s) = 1$ [11]. They have relationship with the Cartesian coordinates (x, y, z) [9,12]. The formula $x^2/(a^2 + \xi) + y^2/(b^2 + \xi) + z^2/(c^2 + \xi) = 1$ represents the concentric ellipsoid surface when $\xi = \text{constant}$. It reduces to the ellipsoid surface when $\xi = 0$.

For the surface satisfies the equation $F(x, y, z) = 0$, the normal vector at the point (x, y, z) of the concentric ellipsoid surface is given as

$$\nabla F(x, y, z) = \frac{2x}{a^2 + \xi}\mathbf{e}_x + \frac{2y}{b^2 + \xi}\mathbf{e}_y + \frac{2z}{c^2 + \xi}\mathbf{e}_z \tag{2}$$

Therefore the unit vector outward normal to the surface of the concentric ellipsoid can be expressed as

$$\mathbf{n} = \frac{\frac{2x}{a^2+\xi}\mathbf{e}_x + \frac{2y}{b^2+\xi}\mathbf{e}_y + \frac{2z}{c^2+\xi}\mathbf{e}_z}{\sqrt{\left(\frac{2x}{a^2+\xi}\right)^2 + \left(\frac{2y}{b^2+\xi}\right)^2 + \left(\frac{2z}{c^2+\xi}\right)^2}} = \frac{1}{H}\left(\frac{x}{a^2+\xi}\mathbf{e}_x + \frac{y}{b^2+\xi}\mathbf{e}_y + \frac{z}{b^2+\xi}\mathbf{e}_z\right) \tag{3}$$

where

$$H = \left[\left(\frac{x}{a^2+\xi}\right)^2 + \left(\frac{y}{b^2+\xi}\right)^2 + \left(\frac{z}{c^2+\xi}\right)^2\right]^{1/2} \tag{4}$$

Then Eq. (1) becomes [9]

$$g_{ox} = Y_{ox} g - \frac{2\rho D}{H}\frac{\partial Y_{ox}}{\partial \xi} \tag{5}$$

Integration is performed over the concentric ellipsoid surface for the above equation. Noting that the coordinate ξ is the only dependent variable of the oxygen mass fraction by symmetry, the char burning rate is obtained as [9]

$$G_c = \frac{8\pi\rho D}{Y_{ox} + \beta}\left(a^2 + \xi\right)^{1/2}\left(b^2 + \xi\right)^{1/2}\left(c^2 + \xi\right)^{1/2}\frac{\partial Y_{ox}}{\partial \xi} \tag{6}$$

where β is the stoichiometric ratio for char combustion.

2.2. Film theory

In industrial combustion apparatuses, there usually exists a relative motion between biomass particles and gas stream. Hence the combustion processes of the biomass char particle involve forced convection. The flow around the particle is no longer symmetric. It has influence on the heat and mass transfer to the particle. The symmetric model is not suitable to this situation. The film theory has been extensively used to handle the char combustion of a fuel particle under forced convection [13]. It is employed presently.

The film theory is a two-stage approach. Firstly, the convective mass transfer to the particle without phase change is assumed to be