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Capillary force and rupture of funicular liquid bridges between three spherical bodies

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ABSTRACT

Capillarity in wet granular materials induces cohesion and increases the material strength due to the attractive forces acting on capillary bridges. In the funicular state, water bridges may be not only formed between two grains but also binding three or more particles, which breaks the axial symmetry of the liquid bridge. This work presents a fundamental study on capillary forces and rupture behaviours of funicular water bridges between three spherical bodies at equilibrium (or static) configurations. Funicular water clusters are numerically solved by an energy minimization approach. Experimental comparisons are made by measuring capillary forces and these confirm the validity of the numerical solutions. Evolutions of capillary forces and rupture distances are investigated systematically by moving two spheres away from the centre. The fixed water volume condition and the constant mean curvature condition are studied respectively. Comparisons are made between the un-coalesced pendular liquid rings and the coalesced funicular bridge. For a same fixed total water volume, the capillary force is weakened by water bridge coalescence to a funicular bridge when the spheres are packed together, but the situation may vary for different contact angles and inter-particle distances. For the constant mean curvature condition, water bridge coalescence does not alter capillary force significantly when particles are packed closely, but the discrepancy is larger by increasing the gap. Funicular water bridge rupture criteria are also proposed based on the studied configurations. It is observed that in general the transmission from pendular to funicular state extends the rupture distance when it has a relatively high water volume or low air-water pressure difference.

1. Introduction

Water presence in granular materials introduces capillary force between particles by water pressure and surface tension and thus significantly increases the material cohesion which keeps the sandcastle standing [1]. With relatively low water content, isolated water bridges are formed between particles and this is referred as the pendular regime. By increasing the water content to the funicular regime, the water bridges may coalesce with each other and one liquid cluster may connect more than two grains. The capillary cohesion change in the funicular state is generally milder than the sharp increase in the pendular state [2–4]. Additionally, the capillary cohesion is usually maximised in the funicular regime [3,5,6] (Fig. 1). Further raise of water content leads the material to the capillary regime as the material is nearly saturated with only entrapped air bubbles, in this state the water surface tension effect vanishes and the sandcastle collapses.

The pressure difference between air and liquid phases is named as the Laplace pressure or suction ($S=u_a-u_w$). Suction changes with degree of saturation (S_r) and the relationship between suction and degree of saturation represents the water retention property [7,8]. Conceptually, low water content corresponds to very high suction and suction decreases rapidly with water content within the pendular regime. Suction changes gently in the funicular state and soon reduced to 0 when the material is nearly saturated (see Fig. 1). The water retention property is not unique and may have a hysteresis effect during drying and wetting cycles [7,9,10].

After the pioneer works of Haines [11] and Fisher [12], the pendular water bridge has been well understood by using two spherical particles with either controlled water volume [13–20] or constant suction [21, 22]. Wet granular materials has also been investigated using the Discrete Element Method (DEM) [23] with the pendular water bridge effects [17,21,24–29]. It is aware from the literature that within the pendular state, the capillary force, which represents the magnitude of inter-particle adhesive effect, and the rupture distance, which determines the total number of interactive pairs, are the main factors for the material strengthening. The X-ray tomography technique has recently been adopted to study the funicular state liquid morphology [4, 30,31]. It is observed that liquid bridge coalescence leads to more complex liquid phase morphologies such as liquid trimer, pentamer and

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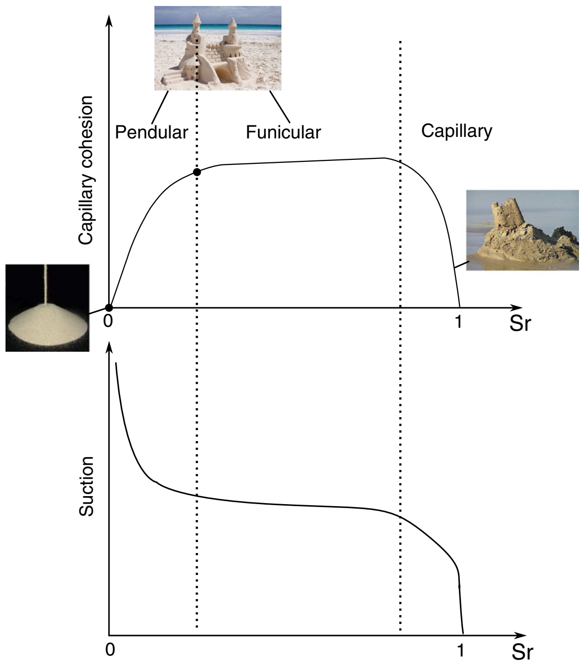

Fig. 1. Conceptual figure displaying capillary cohesion and suction in function of degree of saturation.

tetrahedron [4]. The liquid trimer, as a liquid bridge formed between three particles, is a fundamental structure of the funicular water phase presence. In this case, the study of the capillary force and rupture distance of a funicular liquid bridge between three particles could be a fundamental step toward the understanding of the capillary cohesion in the funicular state.

The two dimensional funicular water bridge effect between three discs can be analytically solved by appropriate simplifications [32,33]. It also has a very limited number of studies on the three dimensional funicular water bridge between sphere spheres [34–36]. Due to the high complexity of the geometry, analytical solutions based on Young-Laplace law are somehow sophisticated. Therefore, a numerical approach based on inter-facial energy minimization can be an alternative choice to study this problem [37]. In this work, the software tool of Surface Evolver (SE) developed by K. Brakke [38] is adopted to minimize the inter-facial energy iteratively. The funicular water bridge effect between three spherical particles is then studied systematically. The solutions are for equilibrium (or static) conditions. The evolution of capillary force and rupture criteria are investigated by lifting two spheres away from a third one steadily with a funicular water bridge formed between them. To study the macro material response under the capillary effect, water content [39–41] or suction [21,42,43] are usually controlled and varied. For this particle scale study, the water volume and suction (or Laplace pressure) are controlled respectively. An experimental test is also carried out to measure the capillary force of a funicular water bridge with various water volumes and to confirm the accuracy of the numerical solution. The effect of the contact angle and the inter-particle distance are demonstrated and rupture criteria based on the numerical solutions are proposed. The coalesced funicular water bridge effect is also compared with the un-coalesced pendular water bridges, which may qualitatively explain the capillary cohesion plateau in the funicular state.

2. Modelling and measurement of funicular water bridge

2.1. Numerical modelling

The funicular water bridge is modelled by using the Surface Evolver software. It is an iterative tool to minimize the total energy includes the surface energy and other body energies subject to various constraints. The surfaces are implemented as triangular facets. The starting liquid shapes can be defined as simplified surfaces and constraints can be applied based on liquid volume, boundary shape, mean curvature, contact angle, etc. By a gradient descent method, the software iteratively evolves the surface shape toward a condition with minimum energy. Surface mesh refinement between iteration processes can improve the solution accuracy.

The gravity effect on water bridges can be assessed by the combination of the normalised liquid volume and the Bond number, and a gravity-free domain has been discussed in [44] for the bridge between two spherical bodies. However, for the funicular water bridge, there is no clear criterion for the gravity effect. Extending the previous criterion to the funicular regime, since the radius of the spheres is relatively small ($R = 0.93$ mm) as well as the water volume used in the experiments we reasonably assumed a low gravity effect. In the following analyses, for simplicity, we will not consider the gravity effect and leave it as a future study. Therefore, without gravity, the interfacial energy will be the main energy to be minimized in the numerical solution. For a liquid bridge connecting N spheres ($N = 3$ in this study, see Fig. 2), the total interfacial energy can be calculated as:

$$E_s = \gamma A^{la} + \sum_{i=1}^{N} \gamma_i^{sl} A_i^{sl} + \sum_{i=1}^{N} \gamma_i^{sa} A_i^{sa} = \gamma A^{la} + \sum_{i=1}^{N} \left(\gamma_i^{sl} - \gamma_i^{sa}\right) A_i^{sl} + \sum_{i=1}^{N} \gamma_i^{sa} A_i \qquad (1)$$

where γ is the liquid-air surface tension, A^{la} is the liquid-air interface area, γ_i^{sl} is the solid-liquid interface tension on the i-th sphere, γ_i^{sa} is the solid-air interface tension on the i-th sphere, A_i^{sl} is the solid-liquid interface area on the i-th sphere, A_i^{sa} is the solid-air interface area on the i-th sphere and A_i is the total solid surface area of the i-th sphere ($A_i = A_i^{sl} + A_i^{sa}$). The solid-air-liquid contact angle on the i-th sphere is noted as θ_i and according to the Young–Dupré equation the following relationship exists:

$$\gamma_i^{sl} - \gamma_i^{sa} = \gamma \cos\theta_i \qquad (2)$$

By knowing the sphere geometry and property (sphere size and γ_i^{sa}), the term $\sum_{i=1}^{N} \gamma_i^{sa} A_i$ becomes a constant (noted as C_1). By substituting Eq. (2) into Eq. (1), the total interfacial energy is:

$$E_s = \gamma A^{la} + \gamma \sum_{i=1}^{N} \cos\theta_i A_i^{sl} + C_1 \qquad (3)$$

The total interfacial energy of an initial surface shape can thus be calculated by the given surface tension γ and contact angle θ_i. Combining with an additional constraint of liquid bridge volume V, the Surface Evolver can help to minimize the total energy iteratively by a new surface shape. It should be noted that the three liquid-solid-air contact lines

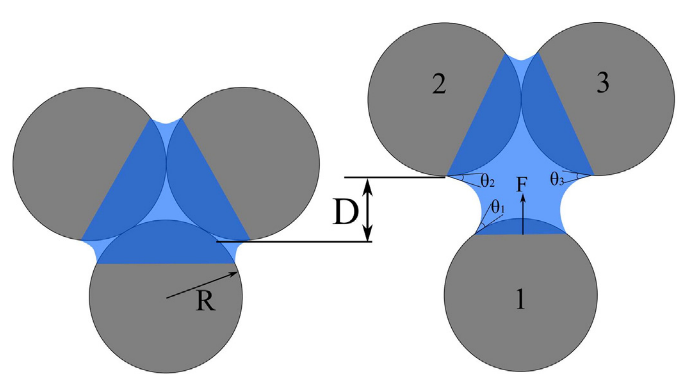

Fig. 2. Configuration of three spheres and the inter-particle distance.