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Localization of unsteady heat source in a tube from pressure measurements with the inverse method

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Abstract

This paper presents work to identify the position of an unsteady heat source in a one-dimensional tube from acoustic pressure measurements with the inverse method. The relationship between the oscillation heat release rate and the pressure can be represented as a Volterra integral equation of the first kind. The discretization method was applied to transform the integral equation into matrix form. To stabilize the solution of the matrix equation, the Tikhonov regularization method was proposed. Experiments were performed to validate the inverse method. A semi-infinite probe system was used to measure the pressure perturbations in the tube, to avoid the high temperature damaging the microphone. Before the pressure measurements were taken, calibration was performed for the semi-infinite probe system to obtain accurate pressure data in the tube. The experiments were performed in three steps. First, the localization of a pure sound source in the tube at ambient temperature was studied. Second, localization of a pure sound source at hot conditions was considered. Third, the pure sound source was replaced with an unsteady heat source, and pressure data were used to determine the position of the unsteady heat source. Results show that calibration and the regularization are both necessary for the determination of the sound source position in the tube. Meanwhile, at the hot and heat release rate conditions, with the consideration of the temperature distribution in the thermoacoustic model, the position of the sound source and the unsteady heat release source can be determined successfully.

Keywords: Thermoacoustic inverse problem; Regularization; Thermoacoustic oscillation

1. Introduction

Thermoacoustic oscillations have been a major technical challenge to the development of high performance propulsion and power generating systems, such as aircraft engines and gas turbines. These oscillations are characterized by strong combustion–acoustic interactions, and can significantly undermine the stability of the combustion systems and increase the pollution emissions. To predict the occurrence of these oscillations, the coupling mechanism between pressure perturbations and unsteady heat release rate needs to be

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understood. One way to monitor the process of thermoacoustic oscillations is the optical method. However, it is difficult to use this method in industrial combustion systems. Therefore, it is necessary to find a non-intrusive method. Fortunately, it is possible to calculate the distribution of heat release rate in the combustion zone from pressure data. Strahle and Ramachandra [1] first investigated the possibility of using the noise radiated from an open flame to recover the heat release rate. A Fredholm integral equation of the first kind was derived to calculate the results. Verification of optical emission measurements showed that it is possible to calculate the fluctuating heat release rate from non-intrusive acoustic measurements. However, the results were very sensitive to noise. Chao [2] extended this non-intrusive method to a gas turbine combustor. An implicit least-square-fit method was developed to solve the Fredholm integral equation of the first kind. He concluded that this method was effective only in certain frequency bands for the sensitivity to noise. Lieuwen et al. [3] reconsidered this problem using a general formulation of the conservation equations. He showed that it is possible to recover the heat release from pressure measurements in a free field or when the oscillations were far from the natural modes of the combustion chamber. At natural frequencies, however, the heat release rate cannot be determined. To solve this problem, Subrahmanyam et al. [4] reformulated this inverse problem as a Volterra integral equation of the first kind. A direct numerical method and implicit least-squares methods were developed to solve the integral equation. They concluded that this reformulation helped in recovering the heat release distribution successfully at all frequencies. Following the work of Subrahmanyam, Kaltenbacher and Polifke [5] applied different regularization methods to the inverse problem to reduce noise sensitivity.

However, the inverse thermoacoustic theory based on the Volterra integral equation did not consider the influence of temperature, which greatly affects the accuracy of the thermoacoustic model. Also, there is not an experimental validation of the theory. To further develop the inverse thermoacoustic theory, this paper proposes an experimental and theoretical study to localize the unsteady heat source from pressure measurements. In this study, an array of microphones was mounted in the axial direction of a quartz tube to measure the distribution of pressure oscillations. To protect the microphone from the heat of the tube, a waveguide system was used for each microphone. Meanwhile, the microphones were calibrated before the experiments. Experiments were conducted at three different conditions: 1) The pure sound source was localized at ambient temperature; 2) The pure sound source was localized at high temperature; 3) The unsteady heat source was localized at condition of thermoacoustic oscillations. Section 2 presents the theory of recovery of the heat release rate from pressure measurements, including the Volterra integral equation that correlates the unsteady heat release rate and dynamic pressure in the tube, and the regularization method. The influence of temperature was also considered in this section. Section 3 presents the calibration of the semi-infinite probe system and the experimental validation of the theory. The influences of the calibration, regularization process and temperature were considered in the analysis. The conclusion is presented in Section 4.

2. Theory

2.1. Volterra integral equation

We consider the one-dimensional situation. The combustor is assumed to contain an inviscid, non-heat conducting ideal gas. The mean flow Mach number is smaller than 0.1. We start from the one-dimensional conservation equations of momentum and energy, in frequency domain, assuming time-dependent harmonic behavior at frequency f. The relationship between the pressure oscillation P and unsteady heat release rate Q can be expressed as a Helmholtz equation,

$$\frac{d^2P}{dx^2} + k^2 P = -\frac{\gamma - 1}{c} ikQ, \tag{1}$$

herein, $k=\omega/c$ is the dimensionless frequency, where ω is the angular frequency of sound, c is the speed of sound, γ is the ratio of specific heats and i is the imaginary unit. Equation (1) can be transformed to a Volterra integral equation by integrating from 0 to ξ and then from 0 to x [4],

$$f[P(x)] = \int_0^x \mathbf{k}(x,\xi)Q(\xi)d\xi, \tag{2}$$

where

$$f[P(x)] = P(x) + k^2 \int_0^x (x-\xi)P(\xi)d\xi - P(0) - xP'(0)$$

is the twice integral of the left side of Eq. (1), and kernel $\mathbf{k}(x,\xi) = -(\gamma-1)ik(x-\xi)/c$. Equation (2) is the Volterra integral equation of the first kind.

2.2. Influence of temperature

Under conditions of thermoacoustic oscillations, the influence of temperature needs to be considered in the wave equation. Assuming the mean flow Mach number is smaller than 0.1, the governing equation can be rewritten as [6,7]

$$\frac{d^2P}{dx^2} + \frac{1}{\bar{T}}\frac{d\bar{T}}{dx}\frac{dP}{dx} + \frac{\omega^2}{\gamma R\bar{T}}P = \frac{-i\omega(\gamma-1)}{\gamma R\bar{T}}Q, \tag{3}$$