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# Finite-Time $\mathcal{L}_2$ Leader–Follower Consensus of Networked Euler–Lagrange Systems With External Disturbances

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***Abstract*—This paper is concerned with finite-time $\mathcal{L}_2$ leader–follower consensus of networked Euler–Lagrange systems in the presence of external disturbances. A distributed finite-time $\mathcal{L}_2$ control protocol is proposed by using backstepping design such that a group of follower agents modeled by Euler–Lagrange systems can follow a desired leader agent and achieve leader–follower consensus in finite time. Moreover, the finite-time $\mathcal{L}_2$ gain is less than or equal to a prescribed value. A simulation example of a network composed of seven two-link manipulators is given to show the effectiveness of the theoretical results.**

***Index Terms*—Euler–Lagrange system, external disturbance, finite-time leader–follower consensus, $\mathcal{L}_2$ stability.**

## I. INTRODUCTION

OVER the last two decades, consensus of multiagent systems has received increasing interest due to its wide applications in formation control of robotic systems [1], [2], power sharing of microgrids [3], and distributed sensing in sensor networks [4], [5]. In general, the existing studies on consensus can be classified into two categories: 1) leaderless consensus [6]–[8] and 2) leader–follower consensus [9]–[13]. The basic principle of leaderless consensus is to design a suitable distributed control protocol based on the information of the agent itself and its neighbors such that all agents can reach an agreement on their states/outputs while leader–follower consensus aims at designing a suitable distributed control protocol such that a group of follower agents can track the leader. We refer readers to the recent surveys [3], [15], [16] on more results concerning consensus of multiagent systems.

Motivated by the fact that the dynamical behavior of some physical systems can be modeled by Euler–Lagrange systems, such as electrical systems and robotic manipulators, a large number of researchers pay attention to consensus of networked Euler–Lagrange systems, which is more challenging as existing results of linear systems fail to deal with Euler–Lagrange systems due to the intrinsic nonlinearity. In [17], synchronization of networked Euler–Lagrange systems was investigated based on sampled data. In [18], a distributed adaptive control was utilized to solve the containment consensus problem of Euler–Lagrange systems. In the case of the leader–follower consensus, a command generator known as the leader provides the desired reference trajectory. All followers are forced to track the trajectory of the leader. Much effort has been made in forcing networked Euler–Lagrange systems to track a desired leader. In [19], the problem of tracking control of networked Euler–Lagrange systems is studied in a switching network by utilizing distributed adaptive control. In [20], sliding mode control was applied to synchronize all following Euler–Lagrange systems with a dynamical leader. In [21], a continuous tracking algorithm with adaptive updating laws of coupling gains was proposed to solve the synchronization problem of leader–follower Euler–Lagrange systems. As the convergence rate is a critical performance index to evaluate the effectiveness of the control algorithm, how to design an appropriate control protocol to improve the convergence rate is important. Finite-time convergence, which ensures consensus to be achieved after a certain period of time, is in great demand. In comparison with asymptotic consensus protocols in [17]–[22], finite-time consensus is more practical, yet more challenging.

Distributed finite-time control algorithms for multiagent systems were proposed in [23]–[25] based on the finite-time stability theory. In [23], leader–follower consensus of linear second-order multiagent systems was considered. An observer-based control algorithm was designed to track a dynamical leader in finite time. However, the result is applicable only for linear systems. Later, a second-order sliding-mode observer was proposed in [24] for a group of Euler–Lagrange systems with a dynamical leader. In [25], the problem of the finite-time

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containment consensus of networked Euler–Lagrange systems with control torque constraints was discussed. On the one hand, the upper bound of the settling time is still unknown in [23] and [24]. On the other hand, the effect of disturbances is not taken in account. In fact, there often exist many different forms of disturbances in practical environments, such as external interferences, stochastic process, and measurement noises. These factors will result in performance deterioration, such as instability, slow convergence rates. It is essential to study the finite-time consensus problem in the presence of external disturbances. In [26], a finite-time bounded controller was designed for delayed conic-type nonlinear systems with mismatched disturbances. In [27], discontinuous and continuous integral sliding mode control protocols were proposed, respectively, to solve finite-time consensus of linear second-order multiagent systems in the existence of bounded disturbances. As adaptive control is an effective way to adjust parameters [28], in [29], the finite-time consensus problem of networked Lagrange systems with external disturbances was solved by using sliding-mode control and adaptive control. While in [30], an adaptive protocol with distributed updating laws was proposed by combining the disturbance compensator technique in a directed network of Euler–Lagrange systems with bounded external disturbances. Effective methods are presented in [26], [27], and [29] to eliminate disturbances, which may suffer the chattering effect. $H_\infty$ control provides us an another option to deal with disturbances. From the perspective of $H_\infty$ performance [31]–[34], backstepping control has been used to study consensus of networked Euler–Lagrange systems with external disturbances [35]. However, these works [33]–[35] restrict their scope with asymptotic consensus. Thus, how to design a suitable distributed finite-time control protocol for networked Euler–Lagrange systems to ensure finite-time leader–follower consensus and attenuate the effect of disturbance at an acceptable level serves as the motivation of this paper.

This paper focuses on finite-time $\mathcal{L}_2$ leader–follower consensus of networked Euler–Lagrange systems in the presence of external disturbances. First, a virtual error variable is defined for every follower based on an auxiliary function including both discontinuous and continuous distributed control protocols. Then in a backstepping framework, the original error systems between the leader and the followers are transferred into systems related with the virtual error variables. By defining a performance vector, finite-time $\mathcal{L}_2$ consensus is analyzed based on the finite-time stability theory. A sufficient condition is derived ensuring leader–follower consensus in finite time. In the meanwhile, the finite-time $\mathcal{L}_2$ gain of the disturbance attenuation is made less or equal than a prescribed value. The upper bound of the settling time is also given. The case without external disturbances is also discussed. Finally, a network of seven two-link manipulators are given to validate the effectiveness of theoretical results.

The remainder of this paper is organized as follows: in Section II, the problem formulation is given and some preliminaries are presented. Section III gives sufficient conditions for finite-time $\mathcal{L}_2$ consensus of networked Euler–Lagrange systems. Section IV presents a numerical example. The conclusion is drawn in Section V.

Throughout this paper, for a given vector $x = [x_1, x_2, \ldots, x_n]^T$ and a scalar $\alpha > 0$, $\mathrm{sig}(x)^\alpha = [|x_1|^\alpha \mathrm{sgn}(x_1), |x_2|^\alpha \mathrm{sgn}(x_2), \ldots, |x_n|^\alpha \mathrm{sgn}(x_n)]^T$, $x^\alpha = [x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha]^T$, and $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^T$, where $\mathrm{sgn}(\cdot)$ denotes the signum function, $\|\cdot\|$ denotes either the Euclidean vector norm or its induced matrix two-norm and "$\otimes$" denotes the Kronecker product.

## II. Preliminaries and Problem Statement

### A. Graph Theory

Consider a group of leader–follower agents labeled by $1, 2, \ldots, n+1$. The agent indexed by $n+1$ is known as the leader and other agents are indexed by $1, 2, \ldots, n$, which are referred to as the followers. Graphs are used to describe an interconnected topology of these $n+1$ agents. The directed graph $G = (\mathcal{V}, \mathcal{E})$ is with a set of nodes $\mathcal{V} = \{1, \ldots, n+1\}$ and edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, where $(i, j) \in \mathcal{E}$ denotes the allowed information flow from node $i$ to node $j$. There are no self-loops in the graph. A path from node $j$ to node $i$ is a sequence of edges, $(j, p_1), (p_1, p_2), \ldots, (p_l, i)$ with distinct nodes $p_k$, $k = 1, 2, \ldots, l$. A graph is connected if there is a path between any of nodes. $A = [a_{ij}]_{n\times n}$ is the adjacency matrix with $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$; otherwise, $a_{ij} = 0$. The set $N_i = \{j \mid (j, i) \in \mathcal{E}\}$ denotes the neighbors of node $i$.

The Laplacian matrix $L = (l_{ij})_{n\times n}$ associated with an adjacency matrix $A$ is defined by $l_{ii} = \sum_{j=1}^{n+1} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$, $i, j = 1, 2, \ldots, n+1$. Then $L$ can be written as

$$L = \begin{bmatrix} L_1 & b \\ 0_{1\times n} & 0 \end{bmatrix}, \quad L_1 \in R^{n\times n},\ b \in R^n.$$

*Assumption 1:* The network of followers is undirected. The leader has a path to every follower.

### B. Problem Statement

Consider a network of multiple Euler–Lagrange systems. The $i$th Euler–Lagrange system is described by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + \delta_i, i = 1, 2, \ldots, n \tag{1}$$

where $q_i \in \mathbb{R}^m$ is the generalized configuration coordinate; $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the inertia matrix; $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{m\times m}$ is the Coriolis/centripetal matrix; $g_i(q_i) \in \mathbb{R}^m$ is the vector of gravitational torques, $\tau_i \in \mathbb{R}^m$ is the vector of control input torques; and $\delta_i \in \mathbb{R}^m$ is a term that includes an external disturbance. For typical mechanical systems, the inertia matrix $M_i(q_i)$ are symmetric positive-definite. It is assumed that $M_i(q_i)$ is bounded, as well as $\delta_i$ for $i = 1, 2, \ldots, n$.

The stationary leader is given by

$$q_{n+1} = p \tag{2}$$

where $q_{n+1} \in \mathbb{R}^m$ is the state of the leader with a constant $p$.

Rewriting (1) in the matrix form yields

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \delta \tag{3}$$