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Applying perturbation technique to analysis of a free piston Stirling engine possessing nonlinear springs

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HIGHLIGHTS

- Multi-scale perturbation method is applied to analyze a nonlinear free piston Stirling engine.
- A systematic analytical scheme is presented to estimate the gas temperature in hot and cold spaces.
- Unique relationships are proposed to predict frequency, phase, pistons strokes, power and efficiency of the FPSEs.
- Validity of the proposed analytical scheme is shown experimentally.

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ABSTRACT

This paper describes a novel design approach of the free piston Stirling engines (FPSEs) based on multiple-scale perturbation method. First, a comprehensive mathematical model for an FPSE possessing nonlinear springs is presented. Then, the method of multiple scales is used to obtain the steady-state response of the engine system. Thus, some useful analytical relationships to predict frequency, strokes of pistons, and phase angle, as well as output power and efficiency of the nonlinear FPSE, are presented. Furthermore, a systematic mathematical approach for estimating the gas temperatures within expansion and compression spaces of the FPSE is proposed based on the perturbation technique. Next, a simulation study is carried out to investigate how much the engine frequency, strokes of pistons, and phase angle of the FPSE are sensitive to the variation of gas temperature. Besides, the effect of changes in the engine design parameters such as mass and stiffness of the pistons on the output power of the FPSE is studied using simulation. Finally, a test engine is developed and experimented to verify the proposed design technique. It is found that the experimental results are in a good agreement with the simulation outcomes of the analytical model through which validity of the proposed design scheme is clearly demonstrated.

1. Introduction

The Stirling engine is an old concept that has received many attentions from the researchers in the recent years owing to its extensive applications in renewable energy technologies. For instance, the Stirling engine is the key element of solar dish power plants as well as combined heat and power systems (CHP) [1,2]; it operates based on a closed regenerative thermodynamic cycle with periodic compression and expansion of a working gas at different temperature levels [3]. The recent interests in the Stirling engine can be attributed to high thermodynamic efficiency, low emission level, multi-fuel capability, and silent operation [4]. The Stirling engines can be classified into two categories, namely, kinematic and dynamic Stirling engines. However, the first Stirling engine invented by Robert Stirling in 1816 was the kinematic type [5]. In the kinematic Stirling engines, the displacer and power pistons are connected together via a linkage mechanism.

In terms of the arrangement of pistons, displacers, and cylinders there are three well-known configurations of the kinematic Stirling engines i.e. Alfa-, Beta-, and Gamma-types [6]. In these configurations, there are at least two moving parts e.g. two pistons or a piston plus a displacer. The phase difference between the motions of moving parts is usually adjusted to 90° through which a maximum output power can be expected based on the Schmidt's model [7]. On the other hand, the dynamic Stirling engines are a more advanced category of the Stirling converters. In the dynamic Stirling engines, the pistons are free to move independently and the motions of the pistons are coupled via the pressure dynamics [8]. The most common types of the dynamic Stirling engines are

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Nomenclature

A	cross-sectional area of the pistons (m^2)
A_r	cross-sectional area of the displacer rod (m^2)
B	damping coefficient ($N\,s\,m^{-1}$)
d	hydraulic diameter (m)
k	thermal conductivity ($w\,s^{-1}\,m^{-1}$)
K	stiffness of spring ($N\,m^{-1}$)
M	mass of piston (kg)
m_g	total mass of gas (kg)
P	linear pressure (Pa)
Q	total heat (J)
q	heat (J)
R	gas constant ($J\,kg^{-1}\,K^{-1}$)
T	temperature (K)
V	volume (m^3)
v	linear velocity (ms^{-1})
t	time (s)
W	work (J)
W_T	power (W)
X	specific heat ($J\,kg^{-1}\,K^{-1}$)
z	position of piston in modal coordinate (m)

Subscript and superscript

C	cold sink
x	position of piston (m)
c	cold gas
d	displacer
E	hot source
e	hot gas
g	gas
loss	heat loss
max	maximum
min	minimum
p	power piston
r	rod
reg	regenerator
0	initial
$'$	nonlinearity

Greek symbols

γ	heat capacity ratio
ε	a dimensionless parameter
η	efficiency
Δ	difference
θ	phase difference
μ	viscosity (Pa s)
ρ	density ($kg\,m^{-3}$)
ω	frequency ($rad\,s^{-1}$)

Fluidyne, thermoacoustic, and free piston Stirling engines. A Fluidyne Stirling engine contains a working gas and either two liquid pistons or one liquid piston and a displacer. Inherent sealing is an advantage of such a Stirling engine. In the thermoacoustic Stirling engines, the mechanical power of the acoustic field is used to generate power [9,10]. Swift [11,12] was among the first researchers who applied the thermoacoustic concepts to thermal engines for power generation. Since, there was no moving displacer in the thermoacoustic Stirling engines they found to be so reliable, efficient, and silent. Besides, if they were designed in a correct way, they could be maintenance free. However, in spite of the recent advances of the thermoacoustic Stirling engines, they are still in the research phase. Free piston Stirling engines are another type of the dynamic Stirling converters. Beale [13] invented the first free piston Stirling engine in 1964. In the FPSEs, the conventional crank mechanisms used in the kinematic Stirling engines were replaced by spring elements. Accordingly, side forces on the piston parts were eliminated and thus, the FPSEs possessed extremely long operating life and high efficiency. Besides, the well-designed FPSEs were usually self-starting and unique in that they eliminated all wearing mechanisms associated with the kinematic Stirling engines and thus, they eliminated the requirement for lubricant [13].

In designing the Stirling engines, it is important to first model the engine system to achieve a high-performance converter and avoid the high cost and time-consuming trial-and-error process. In 1871, Schmidt presented likely the most basic analytical model of the kinematic Stirling engines considering the unrealistic assumptions of perfect regeneration and infinite heat transfer [5]. Since then, many researchers have proposed different mathematical models to justify the engine performance each of which possesses some advantages and drawbacks. In 1977, Urieli and Berchowitz [14] proposed an adiabatic second-order model of the Stirling engines that was a balanced compromise between simplicity, accuracy, and generality for the calculation of the engine performance. They then modified the proposed model taking into account the effect of the imperfect regenerator. Kaushik and Kumar [15] further highlighted the importance of the regenerator efficiency in modeling the Stirling cycle. They reported that the imperfect process of the regenerator significantly influenced the thermal efficiency of the Stirling engines. The first attempt to investigate heat transfer phenomena inside the Stirling engine was likely that by Finkelstein and Organ [16,17]. They proposed an approach to quantify the heat exchange in the regenerator and to consider the compressibility of the working gas in the modeling process. Indeed, considering the imperfect regeneration and the finite heat transfer rate in the Stirling engines considerably affected the gas temperatures in the compression and expansion spaces which resulted in reducing the engine efficiency. Consequently, it is of practical importance to accurately estimate the working gas temperature in a Stirling engine considering the mentioned non-ideal conditions. Rochelle and Grosu [18] considered the effects of exo-irreversibility and regenerator efficiency in the Schmidt model and consequently, presented analytical solutions for engine speed, power, and efficiency for different types of the kinematic Stirling engines. Tavakolpour-Saleh et al. [7] presented a comprehensive mathematical model of the low-temperature Stirling engines with Gamma-configuration considering both the regenerator efficiency and finite heat transfer. Subsequently, they proposed a mathematical scheme to estimate the instantaneous temperature of the working gas in the heat exchangers based on the finite dimension thermodynamic approach. Finally, the validity of the proposed analytical model was demonstrated experimentally. Kongtragool and Wongwises [19] presented eminent works on thermodynamic modeling of the Gamma-type LTD Stirling converters. They showed that the mean pressure power formula was the most appropriate model for calculating the output power of the Gamma-type LTD Stirling engines. Chen et al. [20] investigated the performance of a Beta-type Stirling engine with a moving regenerator via a numerical model. It was shown that the engine power and efficiency were significantly improved using the moving regenerator. Solmaz and Karabulut [21] proposed a mathematical model for a novel config-