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A finite element method for a weakly nonlinear dynamic analysis and bifurcation tracking of thermo-acoustic instability in longitudinal and annular combustors

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HIGHLIGHTS

- Weakly nonlinear approach in a Helmholtz solver framework.
- Single and multiple burners combustion systems are examined.
- Influence of geometry, damping effects and flame characteristics are examined.
- Bifurcation diagrams, supercritical and subcritical, are investigated and discussed.
- The spinning mode, observed at the limit cycle in annular combustor is discussed.

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ABSTRACT

The aim of this paper is to investigate how nonlinear flame models influence the bifurcation process that characterize the transition to self-sustained thermo-acoustic pressure oscillations in gas turbine combustors. The analysis is carried out by means of a Finite Element Method solver able to treat complex combustor systems with multiple burners. The heat release fluctuations are coupled to the velocity fluctuations in the burner by means of nonlinear dependence. Two polynomial expressions of the third and of the fifth order are respectively considered. At first the proposed numerical procedure is validated in a longitudinal configuration against analytical results obtained in a low-order framework. Then, the ability of the proposed numerical approach to treat combustion systems with multiple independent flames is verified on an annular configuration equipped with twelve burners. In both configurations, in order to track bifurcation diagrams, the amplitudes of velocity fluctuations at limit cycles are plotted against the acoustic-combustion interaction index *n* considered as a control parameter. Regardless of the configuration, supercritical and subcritical bifurcations are obtained depending of the chosen flame model. The influence of time delay and acoustic damping is also investigated.

1. Introduction

Lean premixed combustors used in modern gas turbines for power generation and aero-engines are often affected by combustion instabilities generated by mutual interactions between pressure fluctuations (p') and heat release rate oscillations ($\dot{q}'$) produced by the flame [1–3]. Many theoretical and numerical studies are focused on the analysis of limit cycles of low-frequency instabilities considering simple longitudinal configurations assuming theoretical flame response models [4–7]. The main difficulty to perform these analyses is the definition of the model used to describe the response of the flame to acoustic perturbations [8]. If the flame response to acoustic perturbations is modeled by means of a linear Flame Transfer Function (FTF), a stability analysis can be performed in order to identify the frequencies at which the system is unstable [9–12]. However, linear tools cannot account for finite amplitude effects on the oscillation frequency and cannot predict the fluctuations level. These features can be examined by if a nonlinear description of the flame dynamics is available [13,7]. However, the evaluation of a suitable nonlinear flame model for a given combustor is, probably, one of the most challenging task of the study of thermo-acoustic combustion instability. For this reason, it is worth briefly reviewing some recent investigations of

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nonlinear flame models. The first theoretical studies on nonlinear flame response were proposed by Ananthkrishnan et al. [14], Dowling [13] and Lieuwen [15]. On the experimental side, the dependence of gain of the FTF on the input level was measured for the first time by Balachandran et al. [16] and Birbaud et al. [17]. A fully nonlinear Flame Describing Function (FDF) was measured by Noiray et al. [18]. Assuming a weakly nonlinear approach the FDF can be coupled with linear acoustics to predict the nonlinear dynamics of a given system. This approach was proved to be able to predict the stability map of single-burner combustors with a premixed laminar flame [19,20], a premixed swirled flame [21,7] and, also, a diffusive flame [22]. However, a bifurcation analysis at the variation of a control parameter has never been performed with this approach. A nonlinear system can undergo two different types of bifurcation [23]. The supercritical bifurcation shown in Fig. 1(a) is characterized by a gradually increase of the amplitude once reached the bifurcation point (Hopf point). In this condition, all perturbations imposed on the system tend to decay to zero only if the Hopf point is not reached, otherwise all the perturbations reach a new stable periodic solution at the limit cycle equilibrium. The second type of behavior is the subcritical bifurcation (Fig. 1(b)), which is characterized by a sudden increase of the steady state amplitude at the Hopf point. Once reached the limit cycle equilibrium, the perturbations imposed on the system continue to show a stable periodic oscillation even for values of control parameter lower than the one corresponding to the Hopf point, until the fold point is reached after which all perturbations decay to zero. Different numerical techniques have been proposed in order to track the bifurcation diagrams. Subcritical bifurcations are obtained by Moeck et al. [25] performing a systematic variation of parameters and tracking direct time integration. Continuation methods were used in different works, e.g. [14,26,27]. However these methods are computationally very expensive. Theoretical studies have been performed by Juniper [28] and Subramanian et al. [5,29] retrieving supercritical and subcritical bifurcations at the variation of a control parameter. The use of low-order network models to map the bifurcation diagram as a function of a control parameter has been shown by Campa and Juniper [24]. However, the simplification required by theoretical formulations and low-order modeling techniques limit the complexity of the geometries that can be analyzed reducing the industrial appeal on such approaches.

In annular configurations more difficulties arise due to the presence of multiple flames which respond over a wide frequency range [30]. The problem may be simplified by considering only one single azimuthal mode described by Van der Pol oscillator equations coupled with a single nonlinear flame model expressed in terms of pressure perturbation. Results in [31–33] indicate that this approach is able to predict both spinning and standing unstable modes depending on the nonlinearity and nonuniformity in the flame response. Recently, Bourgouin et al. [34] managed to introduce in their analytical one-dimensional framework a more reliable experimental Flame Describing Function (FDF); however, also in this case, the heat release rate from the different burners is considered uniformly distributed over the circumference of the annular chamber. Following this approach, the spinning instability recorded during experiments of the laboratory scale MICCA annular combustor was reproduced in terms of frequency and amplitude of velocity fluctuations at the limit cycle. Multiple independent flames are considered in the acoustic network modeling approach developed by Parmentier et al. [35] followed by Bauerheim et al. [36]. They presented however, only a linear stability analysis of spinning and standing modes. More complex three dimensional geometries with multiple burners can be examined by means of numerical procedures based on the use of the finite element method for solving the Helmholtz equation that models the thermo-acoustic problem. Pankiewitz and Sattelmayer [37], examined a three-dimensional combustion chamber and simulated the growth of the oscillations in the time domain assuming a nonlinear flame model with a saturation mechanism. Campa and Camporeale [10,38] performed a linear stability analysis of a practical annular combustor assuming a distributed flame transfer function in the frequency domain. Very huge computational resources are requested by analysis of onset and growth of instability carried out by means of Large-Eddy Simulation (LES) that allow one to investigate combustion instability by combining pressure oscillations with turbulent combustion phenomena [39,40]. The present article specifically reports a numerical technique able to perform a weakly nonlinear analysis of thermo-acoustic combustion instabilities in a Helmholtz solver framework. The heat release fluctuations are related to the velocity fluctuation in a reference point of the burner by means of a non linear flame models. A third-order and a fifth-order polynomial flame models are considered in this work. The paper shows that the limit cycle condition is predicted solving the damped inhomogeneous Helmholtz equation coupled with a nonlinear flame model. To this purpose, a Fourier analysis is applied to the flame model: it will be shown that the flame can be expressed as the product of a n-τ linear flame transfer function (FTF) by a third-order and fifth-order polynomial expression which saturate the gain of the FTF with increasing amplitude of velocity fluctuations $|\hat{u}/\overline{u}|$. At first the analysis is performed on a longitudinal combustor. Varying the acoustic-combustion interaction index n, the bifurcation diagrams are tracked assuming both nonlinear flame models. Then, the analysis is conducted in an annular combustor with independent flames proving the feasibility of the presented numerical approach also for combustors with multiple flames. In both configurations, the influence of time delay and of the damping level on the limit cycle is also investigated. The thermo-acoustic theory and the description of the flame models used in the study are presented in Section 2. The longitudinal configuration is analyzed in Section 3. Finally in Section 4 results on an annular configuration are reported.

2. The thermo-acoustic problem: Helmholtz solver approach

The derivation of the mathematical model used for thermo-acoustic studies will be briefly discussed in this section. The complete formulation can be found in other works [12,41]. The fluid is regarded as an ideal gas. The effects of viscosity, thermal diffusivity and heat transfer with walls are neglected, the mean pressure is assumed uniform in the domain. The mean flow velocity $\overline{u}$ is assumed much lower than the speed of sound (hypothesis that is generally verified in the combustion chamber of gas turbines [2]). Acoustic losses are included directly into the wave equation as the first order time derivative of the pressure fluctuations p' multiplied by a non-dimensional coefficient ζ [42], index of the acoustic energy that is globally dissipated into a given volume. Under such hypotheses, in presence of heat fluctuations, the inhomogeneous damped wave equation can be obtained [12]

$$\frac{\partial^2 p'}{\partial t^2} + \frac{\zeta c}{\mathcal{L}}\frac{\partial p'}{\partial t} - \nabla \cdot \bar{c}^2 \nabla p' = (\gamma - 1)\frac{\partial \dot{q}'}{\partial t}, \tag{1}$$

where $\dot{q}'$ is the heat release rate fluctuation per unit volume, γ is the ratio of specific heats, ρ is the density and c is the speed of sound. $\mathcal{L}$ is the characteristic dimension of the analyzed system. In this work, $\mathcal{L}$ is assumed equal to the radius in the longitudinal configuration and to $\sqrt{R_{ex}^2 - R_{in}^2}$ in case of the annular combustor, being R_{ex} and R_{in}, respectively, the external and the internal radius of the annulus domain in which ζ is considered. Several approaches have been proposed to experimentally evaluate the global damping coefficient ζ for longitudinal [21,43] and annular [44] configurations. In this work, for each j-th mode the damping coefficient ζ is modeled as