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# A unified framework for model-based multi-objective linear process and energy optimisation under uncertainty

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## HIGHLIGHTS

- A framework for multi-objective linear optimisation under uncertainty is proposed.
- The uncertainty and the multiple objectives are modelled as parameters.
- The optimal solution is expressed as explicit functions of the parameters.

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## ABSTRACT

Process and energy models provide an invaluable tool for design, analysis and optimisation. These models are usually based upon a number of assumptions, simplifications and approximations, thereby introducing uncertainty in the model predictions. Making model based optimal decisions under uncertainty is therefore a challenging task. This issue is further exacerbated when more than one objective is to be optimised simultaneously, resulting in a Multi-Objective Optimisation ($MO^2$) problem. Even though, some methods have been proposed for $MO^2$ problems under uncertainty, two separate optimisation techniques are employed; one to address the multi-objective aspect and another to take into account uncertainty. In the present work, we propose a unified optimisation framework for linear $MO^2$ problems, in which the uncertainty and the multiple objectives are modelled as varying parameters. The $MO^2$ under uncertainty problem ($MO^2U^2$) is thus reformulated and solved as a multi-parametric programming problem. The solution of the multi-parametric programming problem provides the optimal solution as a set of parametric profiles.



## 1. Introduction

### 1.1. Optimisation under uncertainty

Variations in key parameters and data used to mathematically model a system can often lead to unexpected deviation from the predicted behaviour of the system. For example, parameters like raw material quality, machine availability, safety measures and market requirements can fluctuate with respect to time. In energy and process systems, uncertainty can be either epistemic, such as the value of heat transfer coefficient or the kinetic constant of a reaction, or aleatory such as the demand of energy for the next month or the price of raw material used in a process.

To deal with the uncertainty, a number of formulations and solution techniques, including stochastic programming, fuzzy mathematical programming and multiperiod optimisation, have been proposed in the literature [1–5]. In fuzzy mathematical programming, the random parameters are treated as fuzzy numbers, the constraints as fuzzy sets and some constraint violations are allowed. Fuzzy mathematical programming can be either flexible or possibilistic with regard to where the uncertainty is located in the optimisation problem [6]. In the stochastic programming approach, the decision maker has access to probability distributions which describe the nature of the uncertainty. For the case when the distributions are continuous, a discretisation scheme is employed to compute the discrete probability distributions. The deterministic model is then transformed into a multistage stochastic programming problem and a number of scenarios are considered for different realisation of uncertainty [4]. In the two stage stochastic programming approach the optimisation variables are classified in two groups: the first-stage ones which must be determined before the realisation of the uncertainty and the second-stage ones that enact in a recursive way after the value of uncertain

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**Nomenclature**

*Abbreviations*

| | |
|---|---|
| CR | Critical Region |
| DM | Decision Maker |
| LP | Linear Programming |
| MINLP | Mixed Integer Nonlinear Programming |
| $MO^2$ | Multi-Objective Optimisation |
| $MO^2U^2$ | Multi-Objective Optimisation under Uncertainty |
| mp | Multi-Parametric |
| RES | Renewable Energy Sources |
| RHS | Right Hand Side |

*Greek letters*

| | |
|---|---|
| $\phi$ | uncertain parameter |
| $\psi$ | multi-objective parameter |
| $\theta$ | mp-$MO^2U^2$ parameter |

*Letters*

| | |
|---|---|
| $\boldsymbol{F}$ | vector of objective functions |
| $\boldsymbol{h}$ | vector of equality constraints |
| $\boldsymbol{g}$ | vector of inequality constraints |
| $\boldsymbol{x}$ | vector of decision variables |

*Sets*

| | |
|---|---|
| I | set of objective functions |
| X | set of decision variables |
| $\Theta$ | set of mp-$MO^2U^2$ parameters |
| $\Phi$ | set of uncertain parameters |
| $\Psi$ | set of multi-objective parameters |

*Superscripts*

| | |
|---|---|
| $n_x$ | dimension of decision variables |
| $n_\theta$ | dimension of mp-$MO^2U^2$ parameters |
| $n_\phi$ | dimension of uncertain parameters |
| $n_\psi$ | dimension of multi-objective parameters |
| lo | lower bound |
| up | upper bound |

parameters has been realised. Another technique used to approach uncertainty that was initially introduced from Bellman [7], is stochastic dynamic programming, where multistage decision processes are considered and the uncertainty is part of the dynamic scheme. Grossmann and Morari [8], introduced the concept of flexibility analysis to deal with design and operation of process systems. Multi-parametric programming on the other hand, is an optimisation based methodology that provides a complete map of the optimal solution in the entire range of parametric variability [9].

### 1.2. Multi-objective optimisation

A decision maker has to usually deal with a number of objectives to be optimised, for example, cost, environmental impact, energy efficiency, etc. Multi-objective optimisation, offers a well-founded framework for such problems, with a variety of different approaches such as weighted sum method, goal programming and $\epsilon$-constrained methods [10–12]. In the weighted sum method, the decision maker evaluates the relative importance of each objective function with different weighted coefficients and then performs the optimisation by adding the weighted objective functions together. Although this method can be characterised as computationally efficient, since it generates strong non-inferior solutions, the main disadvantages are the difficulty in the determination of the most adequate weighting coefficients for the problem, as well as the fact that it does not guarantee Pareto optimality [13]. In goal programming, one sets targets for all the objectives that appear in the $MO^2$ problem and then seeks solutions that are closest to the target they have already stated, with the objective to minimise the deviation from the goals set. In the $\epsilon$-constrained method, the optimisation is performed for one objective function, i.e. the most preferred one, with the rest of the objectives bounded between appropriate lower and upper bounds [14,15].

In the $MO^2$ framework, a DM solves a multi-criteria optimisation problem, and chooses between different alternatives acting in pursuit of their own choice and as a result, the concept of optimality in $MO^2$ is replaced with what is known as "Pareto optimality". Energy systems are typical examples of systems in which a performance index can conflict with an environmental or financial restriction as seen in the recent work of Luo et al. [16], where the multi-objective scheme was used for the synthesis of utility systems over the financial cost, the environmental impact and the maximisation of the exergy efficiency. A multi-objective optimisation problem was formulated to account for both the environmental impact and the economic efficiency of the system; the authors solved the resulting $MO^2$ problem with weighted sum and $\epsilon$-constrained method. Zhang et al. [17] examined the optimal design of CHP-based microgrids coupled with life cycle assessment analysis.

### 1.3. Multi-objective optimisation under uncertainty

Klein et al. [18], proposed an interactive approach for solving $MO^2$ with uncertainty in the RHS of the technology matrix, based on the concept of mutual efficiency. Kheawhom and Kittisupakorn [19], proposed a two stage algorithm, in which the $MO^2$ problem is solved in the first step with a genetic algorithm and via a stochastic modeller in the second step, where problem decomposition techniques and sequential quadratic programming method are employed to solve the subproblems. Kwak et al. [20] proposed a new method for $MO^2$ under uncertainty problems in energy conservation in commercial buildings, which included heuristics and also insights from human subject studies. An improved multi-objective teaching–learning based technique coupled with stochastic optimisation was proposed by Niknam et al. [21], where the authors deal with the operation of microgrids under uncertainty. A stochastic multi-objective optimisation study for the optimal operation of combined cooling, heating and power (CCHP) systems was presented by in Hu and Cho [22]. The authors considered variations in climate conditions and three different objective functions for the minimisation of operational cost, primary energy usage and carbon dioxide emissions. Recently, Sabio [23] proposed a systematic framework, including a multiscenario stochastic MINLP, in order to handle uncertainty explicitly in $MO^2$ problems for LCA of industrial processes. In their approach even though the uncertainty is considered explicitly, it is modelled as multiple