Simulation Modelling Practice and Theory 66 (2016) 36–49

Contents lists available at ScienceDirect

Simulation Modelling Practice and Theory

journal homepage: www.elsevier.com/locate/simpat

Tuning of adaptive interacting particle system for rare event probability estimation

Damien Jacquemart[a], Jérôme Morio[b,*]

[a] Onera, The French Aerospace Lab, BP 80100, 91123 Palaiseau Cedex, France
[b] Onera, The French Aerospace Lab, BP 74025, 31055 Toulouse Cedex, France

A R T I C L E I N F O

Article history:
Received 12 March 2015
Revised 15 December 2015
Accepted 8 February 2016
Available online 31 March 2016

Keywords:
Interacting particle system
Rare event
Polarization mode dispersion
Credit portfolio losses

A B S T R A C T

The interacting particle system (IPS) for rare event algorithm has been well mathematically formulated, with a wide variety of results on the estimation accuracy of the probability of rare event. Despite this theoretical point of view, the practical side of this algorithm has not been handled completely. Indeed, a tuning parameter has a significant influence on the effective algorithm performance. Moreover, the choice of a good parameter value often proves to be fastidious and may decrease the usefulness of the IPS algorithm. Therefore, we propose a statistical technique in order to make the IPS algorithm fully adaptive. We derive this strategy for threshold exceedance probability estimation and for the estimation of probability density function tail. The performances of the proposed method have been studied on a toy case and on two more complex estimation problems in optical fiber and financial engineering.

1. Introduction

In recent decades, standards of quality and safety requirements are increasingly demanding in numerous industrial and scientific areas. Estimation of probability of rare events has therefore become of great interest, especially in financial engineering [1], air traffic management [2], optical fiber [3], telecommunication networks [4], climatology [5] and high reliability industrial system [6]. However, crude Monte Carlo estimators are no more efficient and specific techniques dedicated to rare events are required.

Here, we suppose that the studied system is modelized by a Markov chain. Such a modelization can sufficiently simulate physical models and its sequential nature makes it flexible to manipulate. That is why such developments are mostly used for risk evaluation in complex dependable systems where numerous algorithms have been studied [7–11]. More precisely, the model under consideration is a Markov chain $\{X_k,\ k = 0, \dots, n\}$. For a fixed positive integer n, it is often of interest to estimate the probability of rare events such as

$$\mathbb{P}(V(X_n) \in A), \tag{1}$$

for V a real valued function and A a subset of $\mathbb{R}$. Without loss of generality, we assume here that the rare event under consideration is characterized by some threshold exceedance of the real valued random variable $V(X_n)$, for some fixed positive

* Corresponding author. Tel.: +33 5 62 25 26 63.
E-mail address: jerome.morio@onera.fr, jerome_morio@yahoo.fr (J. Morio).

http://dx.doi.org/10.1016/j.simpat.2016.02.004

integer n. If B is a real number, we denote the rare set of interest by

$$A_B = [B, +\infty)$$

and the corresponding rare event we wish to estimate its probability stands as follows

$$\{V(X_n) \geq B\}.$$

In the case of a Markov representation of the studied system, the four most popular methods to estimate rare event probability are importance splitting [12,13], extreme value theory [14,15], importance sampling [11,16], and the IPS algorithm for rare event [9]. The IPS algorithm has the advantage on importance sampling not to require another sampling distribution. In fact, even though a wide variety of asymptotic and non-asymptotic results on the accuracy of the rare event probability estimation with the IPS are available in the literature [9,17], the practical side of this algorithm has not been deeply studied. Therefore, we propose a way to facilitate its implementation, and strongly reduce the computational cost.

The idea of the IPS algorithm is to work with a set of trajectories, and to select and multiply at some iteration of the Markov chain the trajectories that are more likely to reach the rare event probability set. However, selecting properly the Markov chain trajectories can require a huge simulation budget. We propose in this paper an efficient adaptive way to easily implement the IPS algorithm making use of an original characterization of appropriate selection functions.

2. Brief review of rare event estimation

2.1. Statistical approach with the extreme value theory

Extreme value theory (EVT) focuses on the behavior of the distribution tail of a real random variable, based on a reasonable number of observations [1,15]. Because of its general applicative conditions, this theory has been widely applied to model extreme risks in meteorological phenomena [18], finance and insurance [1,19] and engineering [14]. This way to proceed is of great interest when one has to work with only a fixed set of data. However, studying the dynamic of the process in the nonextreme regime (rare event probability set not reached) to estimate the rare event probability may lead to biased estimates, especially if the dynamic changes beyond the nonextreme regime.

2.2. Analytical estimation

Theoretical results for the approximation of (1) can be found in the literature and several cases have been successfully studied. Amongst them we can found stationary process [20,21] with application of small set hitting probabilities for Gaussian and Rayleigh processes. Lévy processes, which frequently arise in financial engineering, are studied in [22]. The author of [23] also presents a Poisson clumping heuristic for the estimation of rare event probabilities in continuous time.

2.3. Splitting algorithm

The splitting algorithm has been first proposed in a physical context [24], and a few variants have been then worked out such as in [25–27] and [12]. A summary of the methods can be found in [13,28] and some comparisons in [10]. The most recent advances are given in [29] where the authors proved that the splitting actually fits the framework of the approximation of some Feynman Kac distributions.

2.4. Importance sampling

The idea of importance sampling is to sample the trajectory with another distribution for which the trajectories of the process are more likely to reach the rare event probability set. An introduction is presented [30] and numerous examples are given in [11,31]. The works related to importance sampling in a dynamic framework mainly focuses on finite or denumerable state space models. Examples in that case are found in [32,33] and some techniques are presented in [34–37]. In the special case of finite time T a review of existing methods and asymptotic analysis is presented in [8]. Finally, the case of continuous state space is studied in [38].

2.5. Interacting particle system

This algorithm is originally presented in [9]. The idea of this algorithm is to work with a set of trajectories, and to select and multiply at some deterministic iteration time the paths that are more likely to reach the rare event probability set. This algorithm has the advantage on important sampling not to require another sampling distribution. Some applications are presented in [3,39,40]. The weighted redistribution algorithm precision relies on a good choice of some selection functions. This choice is always replaced by the tuning of a real parameter, say α. The original paper [9] proposes exponential selection functions. This idea is used in [3,39–41]. Authors of the paper [39] proposes other selection functions that still depends on a real parameter.