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# A Puck-based localisation plane theory for rate- and pressure-dependent constitutive modelling of unidirectional fibre-reinforced polymers

Daniel M. Thomson[a,*], Borja Erice[a], Hao Cui[a], Justus Hoffmann[a], Jens Wiegand[b], Nik Petrinic[a]

[a] Department of Engineering Science, University of Oxford, Oxford OX1 3PJ, UK
[b] COMPACT Composite Impact Engineering LTD, 83 Ducie St, Manchester M1 2JQ, UK

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ABSTRACT

A new constitutive model is presented that aims to accurately tackle the effects of multiaxial loading, strain rate and hydrostatic pressure on unidirectional FRPs, while doing so in an intuitive and physically-based manner. In this model, it is assumed that yielding occurs primarily due to shear stresses acting on a critical localisation plane while normal compressive stresses on this plane have a strengthening effect similar to pressure effects in a Drucker-Prager model. In this way, the orientation of the localisation plane, which depends on the stress state of the material, deals with the issue of multiaxiality and the pressure-dependency is accounted for by the Drucker-Prager based yield criterion.

The main appeal of this approach is that a complex three-dimensional problem is reduced to essentially a case of 2D plasticity, which is conceptually very simple and easier to interpret. In addition, this model is supported by a strong physical basis, as it is constructed from the well-established Mohr-Coulomb and Drucker-Prager theories.

A series of experimental results are provided that support the hypotheses on which this theory is based and against which the model has been benchmarked, showing its ability to reproduce the complex observed effects of multi-axial loading, strain-rate and pressure.

## 1. Introduction

Fibre-reinforced polymer (FRP) composites with unidirectionally (UD) reinforced plies, due to their anisotropic nature, present a difficult challenge when it comes to the prediction of damage and failure. Well-known effects in the polymeric matrix materials, such as pressure and rate dependencies, become more complex with the addition of the anisotropic fibre reinforcement. This translates into an extremely complex mechanical response of the composite material resulting from the interaction of all of these individual properties [1–5]. These effects have been well accounted for in various failure criteria [6,7] that not only provide a good correlation between experimental observations and numerical analysis but also provide reasonable physical explanations for the predicted behaviour. However, for accurate and reliable predictions, it is just as crucial to know the correct constitutive relation of the material before the failure criteria are applied or met. Therefore, there should be a similar degree of confidence in the modelling of the non-linear constitutive response of UD FRPs as there currently exists with their failure criteria for the various modes of failure. This means not only being able to predict known experimental results but, also provide a strong enough physical foundation for such a model so that numerical predictions made outside of the ranges tested experimentally can be taken as valid with a reasonable degree of confidence.

### 1.1. Background

In matrix-dominated loading directions UD FRPs exhibit significant non-linear response, similarly to the polymeric matrices that compose them, as has been often reported in the literature [8]. However, due to the heterogeneous structure of the composite, the degree of non-linearity in the stress-strain response varies noticeably under multiaxial loading [1,2,5]. Furthermore, the effects of the hydrostatic pressure as well as the loading rate in the matrix are also translated to the overall macroscopic mechanical response of the composite in a similar fashion. Multiaxial pressure-dependency in the non-linear response and strain-rate effects of UD FRPs has been reported in [3,4] and in [2,5] respectively. Relevant experimental data from these sources has been reproduced below in Fig. 1 and will be used later on in this paper.

It is also worth noting that, while simply referred to here as non-linear behaviour, the experimentally-observed inelastic response can be

* Corresponding author.
*E-mail address:* daniel.thomson@eng.ox.ac.uk (D.M. Thomson).

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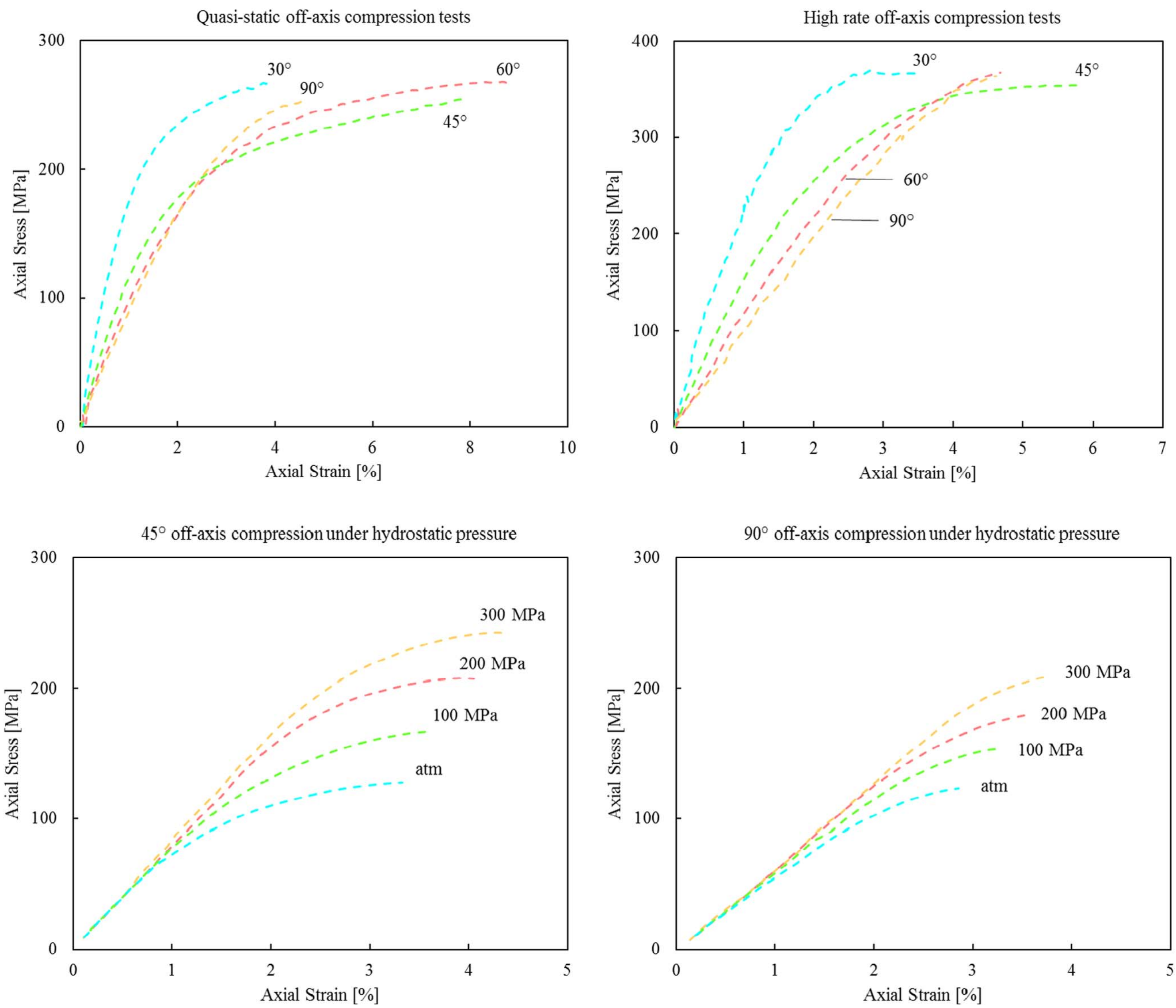


**Fig. 1.** Experimental data reproduced from the quasi-static and dynamic off-axis compression tests in [2] and the 45° and 90 ° off-axis compression tests under hydrostatic pressure in [3], illustrating effects of multiaxial loading, strain rate and hydrostatic pressure.

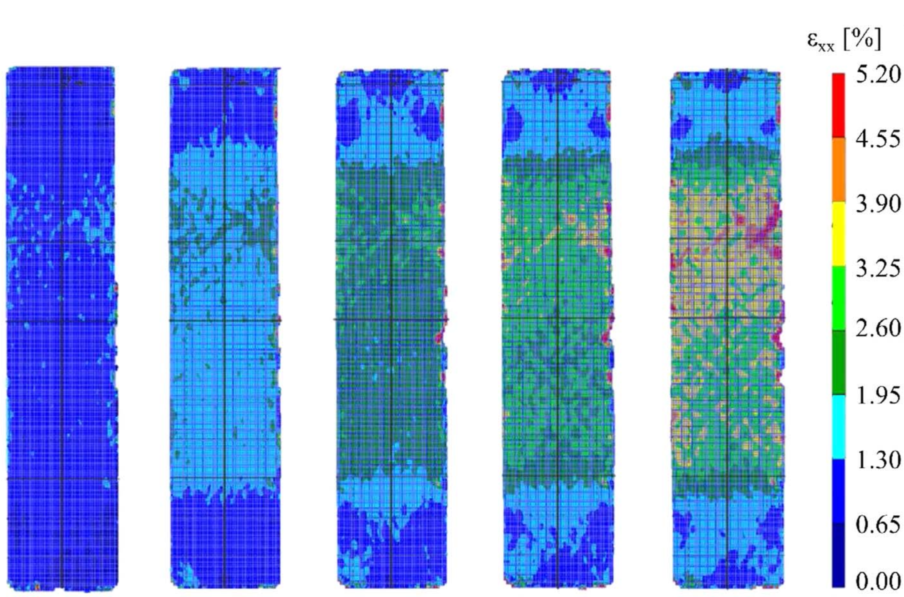


**Fig. 2.** DIC measured longitudinal ($\varepsilon_{xx}$) at different stages during a 90 °off-axis quasi-static compression test carried out in-house on a waisted specimen following the experimental methodology described in [22] for quasi-static tests. From early on in the experiment strain localisation planes can be observed in the top portion of the specimen.