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Free bending vibration analysis of thin bidirectionally exponentially graded orthotropic rectangular plates resting on two-parameter elastic foundations

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ABSTRACT

The vibration of biderctionally exponentially graded orthotropic plates (BEGOPs) resting on the two-parameter elastic foundation is studied. Pasternak elastic foundation (PEF) model is used as two-parameter foundation model. The heterogeneity of the orthotropic exponentially changes depending on the axial and thickness coordinates. The motion equation is derived based on the classical plate theory and solved by using Galerkin method. To validate of current results was made a comparison with the previous studies. The effects of material gradient and orthotropy, and the two-parameter elastic foundations on the dimensional frequency parameters (DFPs) are investigated.

1. Introduction

The wide use of modern composites in various products of modern technology required not only the development of traditional methods for the analysis of thin-walled plates, but also the formulation of new tasks and revealed the need to take into account the new main factors that determine the bearing capacity of structures. Among these factors, anisotropy and heterogeneity of the material occupy an important place. These factors introduce additional complexity into the study of the vibration and stability problems of composite structures. A great contribution to the theory of anisotropic plates was made the work Reddy [1].

Inhomogeneous structures are often used in technical designs that take full advantage of continuous and gradual changes in the physical and mechanical properties of the material. Such structures are widely used in aviation, aerodynamic structure, space vehicles, light-alloy structure of cars and in other engineering structures. Compared to homogeneous orthotropic plates, the adoption of continuous change of material properties can provide important benefits. Indeed, the increase in the number of constructive variables extends the possibilities of advanced composite materials, as well as stability and vibration behaviors may be significantly altered. The reason for the appearance of heterogeneity of the material can be, manufacturing technology, thermal and mechanical treatment, heterogeneity of compositions and a number of other reasons. As a result of the above reasons, the inhomogeneity can simultaneously depend on the spatial coordinates. The basic knowledge on the changes of the material properties is given in the work of Lomakin [2]. Efforts related to the determination of various types of functionally graded anisotropic materials have been the focus of research in recent years [3–6]. Using above mentioned models, several important problems were solved about the oscillations of the functionally graded orthotropic plates [7–12].

In many practical applications, composite plates are in contact with soils or other solid particles and can have significant and unavoidable effect on their behaviors. To correctly determine the influence of the elastic foundation, there are various models, among which one of the effective model was proposed by Pasternak, which is called a two-parameter elastic foundation [13]. Besides, a comprehensive review of elastic foundation models is discussed in the Ref. [14]. The vibration of homogeneous orthotropic plates resting on the two-parameter elastic foundations, which has practical applications in civil, mechanical, marine and aerospace engineers have been studied using various analytical and numerical methods [15–21].

In recent years, the urgency of solving the stability and vibration problems of functionally graded composite plates has increased dramatically. This is explained, first of all, by the continuous expansion of the introduction of inhomogeneous composite plates into load-bearing elements of structures working in contact with different environments. The numerous studies on the vibration of functionally graded orthotropic plates resting on the Pasternak elastic foundation have been published in the literature [22–29]. In the majority of the above mentioned studies, the change in the elastic properties of FG orthotropic

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materials was carried out as the function of thickness or axial coordinates, separately. The main contribution to this study is made by the development and implementation of the vibration analysis for thin exponentially graded (EG) orthotropic plates which the material properties vary depending on the axial and thickness coordinates together and resting on the Pasternak elastic foundation.

2. Formulation of the problem

The configuration of rectangular biderctionally exponentially graded orthotropic plate (BEGOP) with the length a, the breadth b and the thickness h and resting on the Pasternak elastic foundation (PEF) is illustrated in Fig. 1. The plate referred to a system of rectangular coordinate system Oxyz. The mid-plane being $z = 0$ and the origin is at one corners of the orthotropic plate. The x and y axes are taken along the principle directions of orthotropy and z axis is normal to the them. The reaction of the PEF is related to the deflection, w, with the following relationship [13,14].

$$R = K_w w - K_p\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{1}$$

where $K_w(N/m^3)$ and $K_p(N/m^2)$ are spring and shear moduli of the two-parameter elastic foundation [15–27].

It is assumed that the material properties of the orthotropic plate vary in the axial and thickness directions, as follows:

$$E_1 = E_1^0 f_1(X) f_2(Z), \quad E_2 = E_2^0 f_1(X) f_2(Z), \quad G_{12} = G_{12}^0 f_1(X) f_2(Z), \quad \rho = \rho^0 \psi_1(X) \psi_2(Z) \tag{2}$$

where E_1^0 and E_2^0 are the Young's modulus in the x and y directions, respectively; G_{12}^0 is the shear modulus and ρ^0 is the density of the homogeneous orthotropic plate. Furthermore, $f_1(X)$ and $\psi_1(X)$ are exponential functions characterize the change of the Young's and shear moduli, and density in the x direction, respectively; $f_2(Z)$ and $\psi_2(Z)$ are exponential functions characterize the change of the Young's and shear moduli, and density, respectively, in the z direction and the following definitions apply [3–9]:

$$f_1(X) = \alpha_1^X, f_2(Z) = \alpha_2^{Z+0.5}, \ \psi_1(X) = \beta_1^X, \ \psi_2(Z) = \beta_2^{Z+0.5} \tag{3}$$

in which $X = x/a$ and $Z = z/h$ are the dimensionless variables; $\alpha_1 = \frac{f_1(1)}{f_1(0)}$ is the variation parameter of Young's and shear moduli in the x direction in which $f_1(0)$ and $f_1(1)$ are the values of function, $f_1(X)$, on the $x = 0$ and $x = a$ edges of the plate, respectively. $\beta_1 = \frac{\psi_1(1)}{\psi_1(0)}$ is the variation parameter of the density in the x direction in which $\psi_1(0)$ and $\psi_1(1)$ are the values of the function, $\psi_1(X)$, on the $x = 0$ and $x = a$ edges of the plate, respectively. $\alpha_2 = \frac{f_2(0.5)}{f_2(-0.5)}$ is the variation parameter of Young's and shear moduli in the z direction in which $f_2(-0.5)$ and $f_2(0.5)$ are the values of the function, $f_2(Z)$, on the $Z = -0.5$ and $Z = 0.5$ planes of the plate, respectively. Furthermore, $\beta_2 = \frac{\psi_2(0.5)}{\psi_2(-0.5)}$ is the variation parameter of the density in the z direction in which $\psi_2(-0.5)$ and $\psi_2(0.5)$ are the values of the function, $\psi_2(Z)$, on the $Z = -0.5$ and $Z = 0.5$ planes of the plate, respectively. Poisson's ratios of orthotropic material ν_{12} and ν_{21} are constant and the following inequality is satisfied: $\nu_{21}E_1^0 = \nu_{12}E_2^0$.

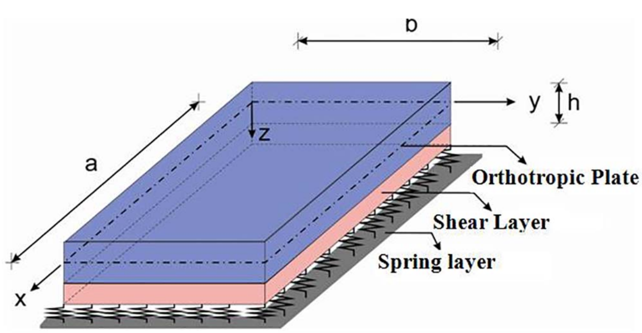

Fig. 1. The rectangular BEGOP on the PEF and the coordinate system.

3. Basic equation

Based on the classical plate theory (CPT), the relationships between the stresses and strains at an arbitrary point of the BEGOPs are written in the following form [2–7]:

$$\sigma_{11} = \frac{E_1^0 \alpha_1^X \alpha_2^{Z+05}}{1-\nu_{12}\nu_{21}}(\varepsilon_{11} + \nu_{12}\varepsilon_{22}), \sigma_{22} = \frac{E_2^0 \alpha_1^X \alpha_2^{Z+05}}{1-\nu_{12}\nu_{21}}(\varepsilon_{22} + \nu_{21}\varepsilon_{11})\sigma_{12} = G_{12}^0 \alpha_1^X \alpha_2^{Z+05}\varepsilon_{12} \tag{4}$$

Let us assume that the Kirchhoff-Love hypotheses are valid for the BEGOPs, and have [1]

$$\varepsilon_{11} = e_{11} - z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{22} = e_{22} - z\frac{\partial^2 w}{\partial y^2}, \quad \varepsilon_{12} = e_{12} - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{5}$$

where e_{11}, e_{22}, e_{12} are the strains in the mid-plane.

The force and moment resultants are expressed by the following relations [1]:

$$(T_{ij}, M_{ij}) = \int_{-h/2}^{h/2} \sigma_{ij}[1,z]dz, \ (i,j = 1,2) \tag{6}$$

Since there are no external forces in the plane of the plate ($T_{ij} = 0$, $i,j = 1,2$) (it assumed that the plate experiences a pure bending), it is therefore assumed that the resultant forces are everywhere equal to zero. In this case, the following conditions can be written:

$$\upsilon_1(e_{11} + \nu_{12}e_{22}) - \upsilon_2(\chi_{11} + \nu_{12}\chi_{22}) = 0 \upsilon_1(e_{22} + \nu_{21}e_{11}) - \upsilon_2(\chi_{22} + \nu_{21}\chi_{11}) = 0 \upsilon_1 e_{12} - \upsilon_2\chi_{12} = 0 \tag{7}$$

where χ_{11}, χ_{22} and χ_{12} are the curvatures of the middle plane and the following definitions apply:

$$\upsilon_1 = \frac{h(\alpha_2 - 1)}{ln\ \alpha_2}, \quad \upsilon_2 = \frac{h^2[2(1-\alpha_2) + (\alpha_2 + 1)ln\ \alpha_2]}{2ln^2\ \alpha_2} \tag{8}$$

Taking into account relations (4), (5) and (7) in the expression (6), we obtain the following expressions for the moments:

$$M_{11} = D_1^0 \Lambda \alpha_1^X\left(\frac{\partial^2 w}{\partial x^2} + \nu_{12}\frac{\partial^2 w}{\partial y^2}\right), M_{22} = D_2^0 \Lambda \alpha_1^X\left(\frac{\partial^2 w}{\partial y^2} + \nu_{21}\frac{\partial^2 w}{\partial x^2}\right) M_{12} = 2D_T^0 \Lambda \alpha_1^X \frac{\partial^2 w}{\partial x \partial y} \tag{9}$$

where D_1^0, D_2^0, D_T^0 are flexural rigidities of the homogeneous orthotropic plate (HOP), Λ is the parameter and the following notations apply:

$$D_1^0 = \frac{E_1^0 h^3}{12(1-\nu_{12}\nu_{21})}, D_2^0 = \frac{E_2^0 h^3}{12(1-\nu_{12}\nu_{21})}, D_T^0 = \frac{G_{12}^0 h^3}{12}, \Lambda = 12\frac{\alpha_2\ ln^2\alpha_2 - (\alpha_2 - 1)^2}{(\alpha_2 - 1)ln^3\alpha_2} \tag{10}$$

Taking into account Eqs. (1) and (2), the partial differential equation of the motion for the BEGOPs on the PEF can be written as [6,8]:

$$\frac{\partial^2 M_{11}}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_{22}}{\partial y^2} - K_w w + K_p\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \rho_1 h \beta_1^X \frac{\partial^2 w}{\partial t^2} = 0 \tag{11}$$

where the following definition applies:

$$\rho_1 = \rho^0 \frac{\beta_2 - 1}{ln\ \beta_2} \tag{12}$$

Substituting (9) into Eq. (11), after elementary transformations we