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A novel uncertainty analysis method for composite structures with mixed uncertainties including random and interval variables

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ABSTRACT

This paper is aimed at proposing a novel uncertainty analysis approach based on polynomial chaos expansion method for composite structures, where mixed uncertainties including normal random and interval variables can be considered simultaneously. Firstly, existing uncertain elastic parameters in composites can be separately dealt with as normal random and interval variables according to the amount of available information and the uncertain response analysis problem with mixed uncertainties is proposed. Then some traditional uncertainty analysis methods are introduced, whose purpose is to point out the shortcomings or deficiencies for given approaches. Furthermore, the basic knowledge for polynomial chaos expansion (PCE) method and a novel uncertainty analysis method on the basis of PCE method for composite structures with mixed uncertainties are explained in details, where the polynomial coefficients are viewed as quadratic polynomial functions of interval variables. Finally, two numerical examples are carried out to account for the validity and feasibility of the developed method. The results show that the developed PCE method can be efficiently applied in uncertainty analysis of composite structures.

1. Introduction

In recent years, advanced fiber-reinforced composites have been increasingly applied in the fields of aviation and aerospace instead of conventional metallic materials due to their superior properties such as high strength, high stiffness and low weight [1,2], etc.. However, in practical applications, similarly to other engineering problems [3–5], it should be recognized that material properties of composite laminates are usually inevitably subjected to a certain amount of dispersions or uncertainties [6,7]. This is mainly attributed to complicated manufacturing processes and their inherent dispersion of constituents. As a consequence, the precise uncertainty propagation analysis for composite structural responses is becoming an important hot topic [8–10].

Generally speaking, uncertainties of composites are usually dealt with as random variables [11–14] in the probabilistic approaches. For example, in the literature [15], based on probabilistic micro and macro-mechanical analysis, the best-fit probability densities for the Weibull, normal and lognormal models are plotted for material properties of fiber-reinforced composite materials. In the literature [16], an experimental characterization of spatial variability in GFRP composite panels is implemented, where tests for goodness-of-fit are performed for the Weibull, normal and lognormal distributions using the methods described in the Composite Materials Handbook [17]. From the evaluated results in the literatures [15,16], it can be concluded that it is more appropriate to model the strength parameters with a lognormal or Weibull distribution and elastic mechanical properties with a normal distribution. But, as the precondition, it always requires that the available information for composite structures is sufficient so that the probability density functions of existing uncertain parameters can be determined precisely. It will decide the correctness and accuracy of further uncertainty analysis and reliability evaluation [18,19]. However, it must be sensed that the known information is always limited for composite laminates in many cases, such as the destructive mechanical property experiments for some developed composites. As a result, the probabilistic approaches cannot be reasonably or appropriately used any more while variation bounds of uncertain parameters can be estimated easily. Furthermore, the non-probabilistic methods are shown to possess great advantages, where uncertain parameters of composites can be modeled as interval or convex-set variables [20–22]. Although it has achieved many successes in these two aspects, there exist at the same time both random and interval variables in some certain cases [23–25], where one can try to transform those uncertain parameters into only one kind of uncertainties in implementing uncertainty propagation analysis. The estimation will be not reasonable and may lead to a wrong result. Therefore, how to carry out a precise uncertainty analysis for composite structures under the mixture of random and

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interval variables is also a worth discussing problem.

On the other hand, in the aspects of uncertain response analysis and prediction of composite laminates, there exist three kinds of commonly-used uncertainty analysis methods including the Monte Carlo simulation (MCS) methods [26,27], first-order expansion methods [28,29], and response surface methods [30,31]. When the MCS methods are widely applied owing to relatively high calculation precision, the time loss is very great so that it cannot be adopted easily for some complex engineering structures with many uncertain parameters. Instead, the first-order expansion method can work well because of its simpleness and time-saving. However, it is only applied appropriately for some linear or hyposensitive nonlinear problems. For strong nonlinear problems of composite laminates, the calculation precision cannot be achieved. That is to say, both the former two approaches have some shortcomings or deficiencies in dealing with uncertain response analysis problems of composite structures. On the contrary, the response surface method can efficiently and precisely approximate the structural responses as a relatively advanced tool. For example, a new hybrid reliability method is developed to compute the interval of the failure probability, where a response surface model is constructed for the limit state functions [32]. A novel approach referred to polynomial correlated function expansion is presented for stochastic free vibration analysis of composite laminates [33], where the proposed approach facilitates a systematic mapping between the input and output variables. Besides, a polynomial chaos based stochastic finite element methodology is developed for uncertainty quantification and failure probability estimation in composite laminated plates [34]. Here, the polynomial chaos expansion (PCE) method is one of typical representations of response surface methods, which can guarantee high calculation accuracy but low cost [35]. In practice, it can be viewed as a response-surface model by taking good use of a set of orthotropic polynomials to precisely represent structural responses and it can combine the Karhunen-Loeve decomposition with polynomial chaos to deal with problems with correlated uncertain parameters. Furthermore, it has achieved many successful applications [36–39]. For example, the non-intrusive Gram-Schmidt polynomial chaos expansion method is adopted for uncertainty propagation of structural uncertainties to dynamic analysis of composite structures, when the parameter uncertainties are modeled as fuzzy membership functions [36]. It is also concluded that the polynomial chaos expansion method can be an effective tool for solving stochastic systems compared to other response-surface methods (RSM) in a concise state-of-art review on surrogate models [37]. Besides, a dynamic uncertain system with random and fuzzy variables is studied using the polynomial chaos expansion method, where the polynomial chaos is viewed as a product of Hermite and Legendre polynomials [38]. When input variables are modeled by probability-boxes accounting for both aleatory and epistemic uncertainty, a two-level meta-modeling approach is proposed using non-intrusive sparse polynomial chaos expansions [39]. As a result, taking good advantages of the polynomial chaos expansion method to deal with uncertainty analysis problems of composite structures with mixed uncertainties may be a better choice.

In view of these reasons, in this paper, it is aimed at developing a novel uncertainty propagation analysis method based on the polynomial chaos expansion method for composite structures with mixed uncertainties including normal random variables and interval variables. The main contribution of this paper is that the developed polynomial chaos expansion method can establish an explicit formulation between mixed uncertain inputs and uncertain structural responses, where the random variables are embedded in the orthotropic polynomials and the interval variables are in the under-determined polynomial coefficients. The remainder of this paper is organized in details as follows. Firstly, according to the amount of obtained uncertainty information, uncertain parameters existing in composite laminates can be separately dealt with as normal random variables and interval variables by virtue of some statistical and non-probabilistic quantification approaches in Section 2. In Section 3, some traditional uncertainty analysis approaches are introduced including first-order Taylor expansion methods for composites with normal random or interval variables and the response surface methods for composites with mixed uncertainties, and some existing shortcomings are pointed out. Furthermore, the polynomial chaos expansion method is explained in Section 4, where the polynomial coefficients of polynomial chaos can be determined through the sparse collocation strategy. Finally, for the sake of solving uncertainty analysis problem for composite structures with mixed uncertainties, a novel method on the basis of the PCE method is proposed in Section 5, where the polynomial coefficients can be expressed as the quadratic polynomial functions of interval variables. Finally, two numerical examples are carried out in Section 6 to demonstrate the feasibility and validity of the proposed method and some conclusion are drawn in Section 7. It shows that the proposed uncertainty analysis method can be efficiently and reasonably applied in the aspects of uncertainty analysis of composite structures with mixed uncertainties.

2. Problem statement

As an advanced engineering material, fiber-reinforced composite laminates are widely applied in the fields of aviation and aerospace. Generally speaking, a composite laminate is a stack of layers of fiber-reinforced laminae, which are made of two different constituent materials including fibers and matrix as shown in Fig. 1. Owing to the fact that the way where fibers and matrix are assembled is very complicated and there inevitably involve a lot of uncertain parameters in material properties of constituents, material properties of a composite laminate are also uncertain in nature. In other words, the elastic mechanical parameters $(E_1,E_2,\nu_{12},G_{12})$ and the lamina strengths (X_t,X_c,Y_t,Y_c,S) can be dealt with as independent random variables or interval variables with limited information by virtue of some statistical and non-probabilistic quantification approaches [40–42]. According to the researches and conclusions of the literatures [15,16], one can know that it is more appropriate to model the elastic mechanical parameters as normal variables and lamina strengths as Weibull or lognormal variables although the tests of goodness-of-fit for the three kinds of distributions can be performed. In this paper, in order to study and discuss conveniently, only the uncertain elastic mechanical parameters are considered and they are treated as independent normal random variables.

When all these uncertain parameters of composites are dealt with as independent normal random variables where the probability density functions can be precisely evaluated with sufficient uncertainty information, they can be written as the following form

$$\boldsymbol{X} \sim N(\mu_{\boldsymbol{X}},\sigma_{\boldsymbol{X}}^2) \text{ and } \boldsymbol{X} = (x_1,x_2,\cdots,x_m)^{\mathrm{T}} \tag{1}$$

Here, $N(\cdot,\cdot)$ denotes the normal random distribution. $\mu_{\boldsymbol{X}} = (\mu_{x_1},\mu_{x_2},\cdots,\mu_{x_m})^{\mathrm{T}}$ and $\sigma_{\boldsymbol{X}} = (\sigma_{x_1},\sigma_{x_2},\cdots,\sigma_{x_m})^{\mathrm{T}}$ are the vectors of mean

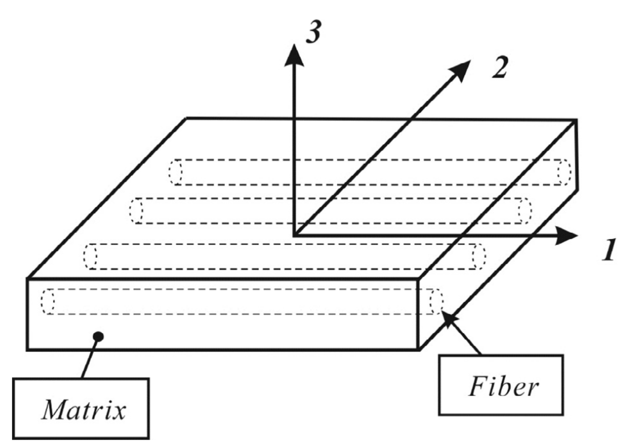

Fig. 1. Sketch map of a typical fiber-reinforced lamina.