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In-plane elasticity of a multi re-entrant auxetic honeycomb

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ABSTRACT

Honeycomb structures are essentially constituted of a repetition of regularly-arranged and loaded substructures. The present study carries out a parametrically investigation of the behavior of a multi re-entrant honeycomb structure with variable stiffness and Poisson's ratio effects. A refined analytical model is specifically developed and compared to full-scale numerical simulations. The analytical model developed is based on energy theorems and takes into full consideration bending, shearing and membrane effects. The influence of the cell walls thickness on the elastic homogenized constants is investigated. The results obtained show a good agreement between the refined analytical approach developed and the numerical computations carried out.

1. Introduction

Cellular materials technology has had a significant development during the past fifty years. Whether they are natural or synthetic, these low density and high specific rigidity materials have seen a widening of their use, in particular during the last twenty years when their production has increased significantly [1,2]. Cellular materials represent an important class of solids that may be used in a variety of engineering applications. Research studies on such systems have been carried out in recent years, particularly about tailored two-dimensional honeycombs [3–7,9]. In most cases, the Poisson's ratio of cellular structures is positive, i.e. the material undergoes a contraction along the direction perpendicular to the one of the load application. However, a negative value of the Poisson's ratio means that the material would laterally expand when stretched, leading to an increase of its volume [8,10,11].

A class of foams that exhibits negative Poisson's ratios has been manufactured and presented for the first time by Lakes [12] back in 1987. The first model of re-entrant structures that shows a negative Poisson's ratio $\nu = -1$ was introduced back in 1985 by Almgren [13]. The structure was first made in 2 D before being extended to 3 D. The model; that may be applied to different geometric structures such as rods, hinges and springs; led to structures that show macroscopic isotropic elastic properties though anisotropic in its microscopic details. Noticing that the molecular dynamics methods with constant pressure or tension displays a fundamental limitation represented by their incapacity to be used to study discontinuous potentials, Wojciechowski [14] applied a constant thermodynamic tension Monte Carlo approach to study the elastic properties of a two-dimensional system of hard cyclic hexamers. His results confirmed the existence of a phase transition between a tilted and a straight phase. He obtained positive results for S_{12} which corresponds to a negative Poisson's ratio. Furthermore, study the elastic properties of a two-dimensional lattice model has been carried out by the same author on triangular lattice hexagonal molecules [15] and shown to display a negative Poisson's ratio at high densities when the anisotropy of the molecules is substantial. Thought using a completely different analysis, the results of Wojciechowski [4] have been achieved by Rothenburg et al. [16] when they interest themselves to a class of microstructures that exhibit a negative Poisson's ratio for large interpenetrations. This behavior is shown to be caused by a greater stiffness of the microstructural elements in shear than in compression. In 1991, Lakes [17] asserted that the Poisson's ratio is governed by aspects of the microstructure identified as the rotational degrees of freedom, the non-affine deformation kinematics or anisotropic structure. Several structures including the chiral microstructure with non-central force interaction or non-affine deformation were examined can also exhibit a negative Poisson's ratio. Geometries that are commonly found in inorganic crystalline materials have

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Notations

A	Wall base.
A^*	Shear section decrease.
E_1, E_2	Young's moduli in directions 1 and 2.
E_s	Young's modulus of basic material.
l	Cell walls lengths.
M	Bending moment.
N	Normal force.
P	Direction 1 concentrated load.
T	Shear force.
t	Cell wall thickness.
U	Elastic strain energy.
u_1, u_2	Displacement in directions 1 and 2.
W	Direction 2 concentrated load.
α	Cell aspect ratio, $(\alpha = \frac{h}{l})$.
β	Wall base aspect ratio, $(\beta = \frac{a}{l})$
γ	γ Wall thickness ratio, $(\gamma = \frac{t}{l})$
ε_1, ε_2	Plane deformation.
θ	Cell internal angle.
ν_s	Poisson's ratio of basic material.
φ	Inclination of the base of the wall.
ν_{12}, ν_{21}	Poisson's ratio in the plane 1–2.

been investigated. A model based on microscopic crystal structures was proposed by Ishibashi and Iwata [18], and resulted in a negative Poisson's ratio. A new mechanism that achieves a negative Poisson's ratio has been developed by Grima and Evans [19]. This model has been based on an arrangement comprising rigid squares joined together at their apexes by joints, can be considered as a two-dimensional arrangement or as a projection on a particular plane of a three-dimensional structure. Triangles were also used and joined together in the same way.

Evans et al. [20] were the first to term such materials as 'auxetic' (from the word "auxetos" that means 'may be subjected to increase'). The diverse analytical models developed to describe the in-plane and out-of-plane mechanical properties are based essentially on the theory of elastic engineering beams, combined to a series of assumptions related to boundary conditions and specific cell walls mechanisms. When two-dimensionally loaded, the honeycomb-shaped cells may be subjected to bending or stretching of their walls, as well as wall the rotations of the connecting junctions (nodes). Several researchers have developed mathematical models based on these mechanisms. Gibson and Ashby [3] and Gibson et al. [21] developed a 2-D model assuming a beam-like bending of the cell walls. Nkansah and Hutchinson [22] however showed that models solely based on bending tend to produce elastic moduli values well over those produced by molecular modeling. In order to improve the bending-based models, Gibson et al. [21] and Masters and Evans [23] incorporated the phenomena of stretching and rotation of the cell walls. Lira et al. [24] describes the out-of-plane shear properties of the multi re-entrant honeycomb configurations. The out-of-plane shear represented by G_{13} et G_{23} affects the transverse deformation of a sandwich panel under a given load level the core providing the deformation contribution via out-of-plane shear, and the face skins via plate bending and tension/compression. In 2013, Pozniak et al. [25] simulated two simple models of two-dimensional auxetic foams. In the first model, the ribs forming the cells of the foam were connected at points corresponding to sites of a disordered honeycomb lattice, while in the second, the connections were not point-like but spatial. Triangles centered at the honeycomb lattice points were used for simplicity. Soft, normal and hard joints were considered for each model respectively corresponding to materials with Young's modulus ten times smaller than, equal to and ten times larger than that of the ribs.

Recently, Li et al. [26] designed a two-dimensional quadrilateral cellular structure made from bi-material strips. Its thermal deformation behaviors were studied via experimental, analytical and numerical approaches. It has been demonstrated that the temperature influences the cell shape and turn it from convex to concave (or vice versa) leading the Poisson's ratio to move from positive to negative (or vice versa). However, the structure proposed in the present work is made of a sole material, is initially hexagonal in shape, and subjected to mechanical stresses. The proposed new cell is modified to become double reentrant leading the structure.

The present investigation tries to highlight the possibility of increasing the precision of the model through designing the novel honeycomb-shaped cell configurations represented in Fig. 1 taking into account the contribution of different stress responses. The analytical model developed is essentially based on the energy theorems along with taking into consideration the shearing and membrane impacts. It is an extension of a previous studies solely based on bending.

2. Theoretical model

An initial analytical model based on bending deformations only of the ribs has been presented in [5]. To take into account the strain energy associated to the shear and normal forces two different loadings are considered in this paper: one along the vertical direction, and the other on the horizontal direction. They are noted 1 and 2 respectively in Fig. 2. The analytical model developed is essentially based on the theorem of Castigliano; the honeycomb cell walls are considered as beam elements and simultaneously subjected to the three types of loading – bending, membrane and shear (Fig. 2a).

The strain energy for the three deformation mechanisms is expressed by:

$$U = U_M + U_N + U_T = \int_0^l \left(\frac{N^2}{2EA} + \frac{M^2}{2EI} + \frac{T^2}{2GA^*}\right) dx \quad (1)$$

According to the Castigliano's theorem, the displacement of a beam under the influence of a force P may be expressed as:

$$u = \frac{\partial U}{\partial P} \quad (2)$$

2.1. Direction 1

For a solid subjected to a linear elastic deformation the energy theorem formulates the strain energy of the three beam elements of the unit cell as a function of the concentrated load P and bending moment M:

$$\left\{P, M = \frac{P}{2}[L\sin(\theta) + 2a\sin(\phi)]\right\} \quad (3)$$

The displacement of the beams system under the concentrated load P can be expressed in the case of bending as:

$$u_1^M = \frac{1}{2}\frac{P}{E_S b}\left(\frac{L}{t}\right)^3 [1 - \cos(2\theta) - 8\beta^3\cos(2\phi) + 6\beta\cos(2\theta) - 12\beta^2\cos(\theta+\phi) + 12\beta^2\cos(\theta-\phi)] \quad (4)$$