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Effect of selected elements of the coupling stiffness submatrix on the load-carrying capacity of hybrid columns under compression

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ABSTRACT

The problems of a multi-mode buckling approach which is based on Koiter's theory of a hybrid column are presented in this paper. An interaction of global buckling modes with the local ones is discussed. There are many different local and global buckling modes. Their selected combinations are dangerous and cause a reduction in the load-carrying capacity. All walls of the hybrid column were plane and made of many layers. The outer layers were thermal barriers and made of a TiC ceramic layer or an AL-TiC-type FGM. The inner layers were composed of aluminium layers and a few carbon-epoxy laminate layers. The classical laminate theory is used to define the ABD matrix which described the relations between applied loads and the associated deformations. The layup configuration of the hybrid column is general, so the coupling submatrix B is non-trivial. This submatrix has a significant impact on the value of local buckling load, whereas its effect on the value of global buckling load can be neglected. The main topic discussed in this paper is whether and how individual elements of the B submatrix can change the load-carrying capacity of a hybrid column. A detailed discussion is conducted for simple supported columns with opened cross-sections subjected to mechanical loads only. Thermal effects are neglected.

1. Introduction

The present paper is the last part in a series of papers prepared by Kolakowski Z., Mania R.J. and Teter A. [1,2]. The main objective of these papers is to discuss an influence of the coupling stiffness submatrix B on the post-buckling behaviour of thin-walled columns made of Functionally Graded Materials (FGM) and/or Fibre Metal Laminates (FML) subjected to compression. Columns with closed [2] and open [1] cross-sections were discussed. The geometrical dimensions of the columns were chosen in such a way that a strong interaction between buckling modes occurred. In this case, different types of global and local buckling modes can be found. There are numerous combinations of buckling modes which have to be checked to determine the load-carry capacity of the structure. The applied multi-mode buckling approach based on Koiter's theory makes it possible.

The ABD matrix was calculated with the classical laminate theory (CLT). Each wall of the column was divided into many layers. The mechanical properties of each layer were known. Because the sequence of layers was not symmetrical, the coupling stiffness submatrix (denoted as B) was non-trivial [3]. This matrix is very important to determine the value of local buckling stresses or the load-carry capacity. A detailed analysis was carried out for the pre-defined configuration of the layers to impose the B submatrix form. It is particularly interesting which elements of the B submatrix can improve and which ones can worsen the post-buckling behaviour of thin-walled columns with open cross-sections subjected to compression. This problem will be discussed in detail further in the paper. Variant I for examples I and II, cases 1 and 2 from [1] is adopted in the present study. In these cases, the B submatrix has no trivia elements except for $B_{16} = 0$ and $B_{26} = 0$.

The state of the art in interactive buckling of thin-walled plate structures made of different types of materials is discussed in authors' papers (e.g., [4]). More details on interactive buckling of FGM structures can be found in [3,5,6] and for FML structures in [7,8], respectively. Finally, papers [1,2] contain a literature review as far as FGM/FML structures are concerned. Additional information about structures made of Functionally Graded Materials under different types of loads is presented in review papers by Liew et al. [9], Jha et al. [10], Swaminathan et al. [11] or a monograph by Hui-Shen [12]. A more extensive literature survey is to be found in [1,2], whereas a multi-mode buckling approach based on Koiter's theory

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was discussed in monographs written by Thompson and Hunt [13], van der Heijden [14] or Kubiak [15].

2. Formulation of the problem

In the case of a thin-walled column, an effect of the interaction between various buckling modes has to be taken into account to determine its load-carrying capacity. The strongest interaction can be obtained between global and local buckling loads. The buckling modes have to be selected to attain the lowest value of the load-carrying capacity [1,2], keeping in mind that the interaction between local buckling modes is very weak.

To solve the problem of the interaction between various buckling modes, a multi-mode buckling approach based on Koiter's theory was applied. A thin-walled column was divided into plate elements. A system of differential equilibrium equations was derived with the variational method. It was solved with the asymptotic perturbation method. Details can be found in [1–6,15].

In the present study, a four-mode buckling approach is applied. The flexural global mode (Euler), the flexural-torsional global mode, the symmetric local mode and the antisymmetric local mode are taken into consideration. In this case, the non-linear system of equations can be written as [1–6,15]:

$$\left(1-\frac{\sigma}{\sigma_i}\right)\zeta_i + a_{pqi}\zeta_p\zeta_q + b_{iiii}\zeta_i^3 - \frac{\sigma}{\sigma_i}\zeta_i^* + \ldots = 0 \quad \text{for } i = 1,2,3,4 \tag{1}$$

where: σ_i, ζ_i, ζ_i^* – buckling stress, dimensionless amplitude and dimensionless amplitude of the initial imperfections corresponding to the first, second, third, fourth buckling modes, respectively; σ – compressive stress; a_{pqi} and b_{iiii} – constant coefficients. All coefficients were found on the basis of Koiter's theory which was presented in papers by Kolakowski [16,17]. The range of indices p, q is from 1 to 4. The summation is done on the repeated indices. The load-carrying capacity was determined for the structure with an initial geometrical imperfection [1,18]. It is the maximum value of the compressive stress corresponding to the limit point. In this case, the Jacobian of the system of Eq. (1) is equal to zero.

In the present paper, multilayer columns with top-hat or lip channel cross-sections subject to mechanical compression only are under investigation [1]. They were simply supported. The initial geometrical imperfections had the form of the eigenbuckling modes which were chosen in Eq. (1). Their amplitudes were not equal to zero. The constitutive law for the multilayer material was obtained with the classical laminate theory (CLT) [19]. The configuration of the layers was not symmetrical, so the coupling stiffness submatrix was not 0. In this case, strong coupling effects can be observed. The main issue discussed in this paper is whether and how individual elements of the coupling stiffness submatrix B can change the load-carrying capacity of thin-walled columns. This submatrix has a significant impact on the value of local buckling load, whereas its effect on the value of global buckling load can be neglected. It is thus possible to increase the load-carrying capacity in this type of structures.

3. Analysis of the results

Detailed analyses were performed for the following cross-section dimensions of hybrid columns (Fig. 1): b_1 = 300 mm; b_2 = 150 mm; b_3 = 50 mm and the thickness $t_{T1} = t_{T2} = t_{T3}$ = 4.4 mm [1]. A top hat cross-section (Fig. 1a) and a lip channel cross-section (Fig. 1b) were considered [1]. The length of all columns was the same and equal to 4500 mm. Two different configurations of three sublayers (i.e., TiC ceramics, Functionally Graded Materials and Fibre Metal Laminate) were considered. The first one was $[TiC/FGM/FML]_T$ and was denoted as CM. In this case, the first outer sublayer (Fig. 1) was ceramics (i.e., TiC). The second configuration was $[FML/FGM/TiC]_T$ and was denoted as MC. The first outer sublayer was FML.

The FML sublayer is made of an alternate sequence of five layers of aluminium (aluminium is always the outer layer) and 4 layers of the prepreg, with the total thickness equal to t_1 = 3.0 mm and the following characteristics [1]:

- aluminium (denoted as Al) – Young's modulus: 69 GPa, Poisson's ratio: 0.3 and the thickness 0.4 mm;
- prepreg (denoted as P) – Young's modulus: 30.75 GPa, Poisson's ratio: 0.144 and the thickness 0.25 mm.

The layup configuration of the FML sublayer was $[Al/P/Al/P/Al/P/Al/P/Al]_T$.

The FGM sublayer of the thickness t_2 = 1.0 mm was made of Al-TiC [1]. The volume fractions of ceramics V_c and metal V_m were described as usual with a simple power law of a distribution throughout the structure thickness [1]:

$$V_c(z_2) = \left(\frac{z_2}{t_2}+\frac{1}{2}\right)^q \underbrace{=}_{q=1} \left(\frac{z_2}{t_2}+\frac{1}{2}\right) \tag{2}$$

$$V_m(z_2) = 1 - V_c(z_2) \tag{3}$$

where: $-t_2/2 \le z_2 \le t_2/2$, t_2 – thickness of the FGM sublayer, z_2 – coordinate describing the thickness of the FGM sublayer, and the indices m and c refer to the metal and ceramic material constituents (Al-TiC), respectively. The material properties are: Al Young's modulus: 69 GPa, TiC Young's modulus: 480 GPa, Al Poisson's ratio: 0.33, TiC Poisson's ratio: 0.20. That sublayer was modelled with 20 composite layers [1].

The TiC sublayer (i.e., ceramics) was made of TiC. It was composed of one composite layer with the assumed thickness of the ceramic layer t_3 = 0.4 mm [1].

The total thickness of each plate of the columns under consideration was $t_T = t_{T1} = t_{T2} = t_{T3} = t_1 + t_2 + t_3$ = 4.4 mm (like Variants I in [1]).

Thus, the obtained four different cases are marked as: TH-CM, TH-MC and LC-CM, LC-MC, where: TH – column with a top hat cross-section, LC – column with a lip channel cross-section, CM – $[TiC/FGM/FML]_T$ sub-layup configuration, and MC – $[FML/FGM/TiC]_T$ sub-layup configuration, respectively. In all the cases under analysis, the load-carrying capacity was determined, assuming that the coupling submatrix B had various element composition. The applied analytical-numerical method (ANM) [1,2] allows for numerical setting to zero selected elements of this submatrix. Such a procedure enables one to track an influence of particular elements of this matrix on buckling response. Four different variants were considered. Variant I corresponds to the submatrix B determined for the assumed layer arrangement of column materials (i.e., $B_{16} = B_{26} = B_{61} = B_{62} = 0$, and the remaining coefficients $B_{ij} \neq 0$ for i, j = 1, 2, 6). This case was discussed in Refs. [1,2]. In Variant II, all elements of the matrix B are zero (i.e., $B_{ij} = 0$ for i, j = 1, 2, 6). In Variant III, it was assumed that $B_{11} = B_{22} = B_{12} = B_{21} = 0$, whereas in Variant IV – only $B_{66} = 0$. Moreover, for all the variants of columns under consideration in this study, the following elements of the stiffness matrix satisfied the following relationships:

$$A_{16} = A_{26} = D_{16} = D_{26} = B_{16} = B_{26} = 0 \tag{4}$$

$$A_{ij} = A_{ji} \tag{5}$$

$$D_{ij} = D_{ji} \tag{6}$$

$$B_{ij} = B_{ji} \tag{7}$$