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Nonlinear laminated plate theory for determination of third order elastic constants and acoustic nonlinearity parameter of fiber reinforced composites

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ABSTRACT

The present article reports on the development of a nonlinear laminated plate theory for fiber reinforced composites. The model uses material nonlinearity, i.e. nonlinear stress-strain relationship to describe the effective second (C_{ij}) and third order (C_{ijk}) elastic constants for a laminated plate. Since each lamina can have different fiber orientation, the rotation of the second and third order stiffness matrices were also incorporated into the model. Theoretical results for variation of third order elastic constants and the acoustic nonlinearity parameter with rotation angle have been presented. To validate the laminated plate theory, Nonlinear Resonant Ultrasound Spectroscopy (NRUS) experiments were carried out on seven different laminate systems with different fiber orientations and laminate sequences. Literature values for the elastic constants were used to predict the acoustic nonlinearity parameter and third order elastic constant. These were further compared with experimentally determined values, and a good agreement in the trend was observed. Since there are several combinations of fiber orientation and laminate sequence, the present theory will be helpful to determine the effective nonlinear properties of any given laminate system.

1. Introduction

The concept of higher order elastic constants has been well explored in the field of nonlinear elasticity of solids. This has gained prominence in the recent history due to its wide applications in material science, biomechanics, and more. While linear theories of elasticity can describe the response very well for small deformations and strain amplitudes in vibration, they tend to break down when it moves into large deformation and finite strain amplitude regimes. This required new theories which can model the response accurately across wide strain ranges (low to high range). In general, nonlinear elasticity theories of solids can be split broadly into geometric nonlinearity, material nonlinearity and combination of both. Geometric nonlinearity describes the nonlinear relationship between strain and displacement, while material nonlinearity describes the nonlinearity of stress-strain relationship. Based on the material and physical conditions, both descriptions have been used to model the material response. Rubber for example is a material which can undergo large deformations and whose nonlinear characteristics are well explored in the literature. Such materials are strongly nonlinear, and therefore have been studied carefully. In contract, most engineering materials such as copper, aluminum, composites etc. are weakly nonlinear (material nonlinearity) and their nonlinearity can be more difficult to characterize.

The concept of higher order elastic constants comes from the Taylor series expansion of the strain energy density to accurately model the nonlinearity of the material response at higher strain amplitudes [18]. Note that this is still in the elastic region and well before the plastic region. There are other studies which investigate the nonlinearity arising from plasticity [36] and damage [9,38] in composites as well. The coefficient of the zeroth order strain term is called the second order elastic constants (SOEC) and the higher order coefficients of the higher order strain terms, $n = 2$, 3, 4 etc. are called the higher order constants, namely third order elastic constants (TOEC), fourth order elastic constant (FOEC), and so on. SOEC have been traditionally used to describe the mechanical properties of the material, i.e. for an isotropic material, the Young's modulus, shear modulus and Poisson's ratio, or the Lame constants; λ and μ. However, with increased interest in the nonlinear properties of the material, the TOEC and FOEC will eventually be utilized similar to the SOEC. Mathematically, they are coefficients of a Taylor series expansion, however, they possess a much deeper physical meaning. The higher order constants for crystals can be determined theoretically using empirical force-constant models,

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molecular-dynamics simulations, and first principles total-energy methods using density function theory (DFT) [43] [15]. The TOEC are especially important because they are dependent on various lattice parameters such as interatomic potential, ion-electron pseudopotential etc. [15]. At an atomistic level, they describe the lattice anharmonicity, which is defined as the anharmonic response of lattice vibration. TOEC have also been shown to be more sensitive to changes in material microstructure [6]. Although the TOEC can be determined theoretically for crystalline structures, obtaining theoretical higher order constants from first principles for heterogeneous structures such as fiber reinforced composites which are mixtures of two dissimilar materials can be challenging. Since each layer can have a set of TOEC, the nonlinear response of the entire laminate or layered structure will be controlled by effective TOECs. This will be essential for laminated structures such as metallic plates with polymer coatings, multi-material laminated armor and composite materials. Focusing on composite materials, extensive work has been done on experimental characterization of the nonlinear properties of composites [12,17,2,7,42]. Most of the nonlinear studies deal with shear properties and characterize the higher order shear constants. From a modeling perspective, the use of nonlinear hyper-elastic material models, such as the Ramberg-Osgood model [21,4,10,24], and the Ogden model [27,33] are well known for composites. The idea of deriving the constitutive nonlinear stress-strain relationship from the strain energy function is well known for crystalline and metallic structures. This was first used for composites by Hann and Tsai [12], who described the system with only the fourth order shear properties. There are several ensuing works which utilized the hyper-elastic model for nonlinearity, but most were limited to metallic structures. More recently the works by Prosser et al. [30] and Elmore at al. [11] are considered the most comprehensive work for composites. Rauter et al. [31] had used the hyper-elastic Murnaghan's material model [25] to study cumulative second harmonic generation of a propagating guided wave and eventually applied it for fatigue monitoring of composite structures.

The objective of the present work is to build on the strain energy model for nonlinearity prescribed earlier and apply it for laminated composites. Each individual layer of fiber reinforced composite is orthotropic, and the relative fiber orientation will give rise to a different set of linear and nonlinear elastic constants. There is only one instance where the entire set of SOEC and TOEC of carbon fiber reinforced composites have been measured [30]. Also, the acoustic nonlinearity parameters which can also be used to obtain the higher order constants has been measured accurately only once [11]. There are several techniques which can be used to experimentally measure the TOEC. The well-known technique in composites community is static testing followed by curve fitting to obtain nonlinear coefficients [20]. The most commonly used technique in the dynamics community is acousto-elasticity [16] followed by finite amplitude waves [37] [5], and other techniques such as collinear wave mixing [23], coda wave interferometry [28], dynamic acousto-elastic (DAE) [32]. More recently, nonlinear resonance ultrasonic spectroscopy (NRUS) [40,8] has also been used to measure TOEC. Among these techniques, acousto-elasticity and finite amplitude waves are considered as benchmark techniques, and the rest are considered non-traditional techniques with each having its own advantages and disadvantages. The NRUS (and DAE possibly) in particular can be used for measuring at least one TOEC in relatively thin structures that cannot be tested using acousto-elasticity or finite amplitude waves.

The present work aims to develop a nonlinear laminated plate theory using nonlinearity elasticity, and determine the effective TOECs of a laminated structure. A nonlinear theory which includes the elastic material nonlinear properties of each lamina of a laminated structure, and predicts the effective linear and nonlinear properties has not been explored in the literature. Since such a theory will be applied to fiber reinforced composites, the effect of fiber orientation, i.e. rotation of the SOEC and TOEC orthotropic stiffness was also accounted for. The influence of fiber orientation on the SOEC, TOEC and the acoustic nonlinearity parameter (β) was studied using their analytical relationships. Finally, to validate the nonlinear laminate theory, seven laminate systems were chosen and NRUS experiments were carried out to determine the effective TOEC and SOEC. Theoretical values for TOEC and SOEC were used from a previous study to compare against the experimental results and a good agreement in trend was observed.

2. Nonlinear laminated plate theory

In classical elasticity, the strains are assumed to be small and thus the strain energy function is a homogenous quadratic function of strains. However, when the strains are not infinitesimal, the higher order terms will begin to influence the strain energy function [18] which is given by:

$$\varphi = \varphi + c_{ij}\eta_{ij} + \frac{1}{2}c_{ijkl}\eta_{ij}\eta_{kl} + \frac{1}{6}c_{ijklmn}\eta_{ij}\eta_{kl}\eta_{mn} \tag{1}$$

where η_{ij} are the Lagrangian strain components and c's are the material constants [3]. If the initial energy and cubical dilation of the body are zero, then the first two terms of Eq. (1) can be neglected,

$$\varphi = c_{ijkl}\eta_{ij}\eta_{kl} + c_{ijklmn}\eta_{ij}\eta_{kl}\eta_{mn} \tag{2}$$

c_{ijkl} are the second order elastic constants of the material, and it is a fourth order tensor with 81 constants. It can be reduced to 21 independent constants for a triclinic material, and can further be reduced using symmetries. For an orthotropic material, it reduces further to 9 constants, for transversely-isotropic material; 6, and finally for isotropic materials; 2. The c_{ijklmn} are the third order coefficients and it is a sixth order tensor with 729 constants, which reduces to 56 for triclinic material, 20 constants for orthotropic material, and 3 for isotropic materials [14]. Using Voigt notation, the constant indices can be rewritten as: $c_{ijkl} \rightarrow C_{IJ}$ and $c_{ijklmn} \rightarrow C_{IJK}$, with ij = 11, 22, 33, 23, 31, 12 → I = 1,2,3,4,5,6.

Using the strain energy definition, the first Piola-Kirchhoff stress tensor can now be written as [13]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + M_{ijklmn}\varepsilon_{kl}\varepsilon_{mn} \tag{3}$$

where, M is given by:

$$M_{ijklmn} = C_{ijklmn} + C_{ijln}\delta_{km} + C_{jnkl}\delta_{im} + C_{jlmn}\delta_{ik} \tag{4}$$

δ is the Kronecker-delta function.

2.1. Nonlinear lamina analysis

In fiber reinforced composites, a continuous lamina is defined as a single layer of fibers arranged together so that the fibers are parallel. An assembly of several lamina or layers is called as a continuous fiber laminate. A global coordinate is defined (x,y,z) for the plate, and a local coordinate which coincides with the fiber orientation is defined as (1,2,3) as shown in Fig. 1. The generalized stress-strain relationship in Eq. (3) can be reduced to the x-y (or 1–2) plane since the lamina are very thin and don't support σ_z (or σ_3). The various stress in the 1–2 plane are shown in Fig. 1. Expanding the stress-strain relationship:

$$\begin{aligned}\sigma_{11} = {} & C_{11}\in_{11} + C_{12}\in_{22} + C_{16}\in_{12} + M_{111}\in_{11}\in_{11} + 2M_{112}\in_{11}\in_{22} \\ & + 2M_{116}\in_{12}\in_{11} + M_{122}\in_{22}\in_{22} + 2M_{126}\in_{22}\in_{12} \\ & + M_{166}\in_{12}\in_{12}\end{aligned} \tag{5}$$