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Free-edge stress fields in cylindrically curved symmetric and unsymmetric cross-ply laminates under bending load

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ABSTRACT

In this paper, an analysis method for the determination of displacement, strain and stress fields in cylindrically curved cross-ply laminates under bending load is presented. The considered cross-ply laminates may be either symmetrically or unsymmetrically laminated and are clamped at one end while the other end is loaded by an evenly distributed bending moment. The analysis method employs a layerwise plane strain approach in the inner regions of the laminate in which the stresses in each layer are represented by adequate formulations for Airy's stress function. In the regions of the free laminate edges where significant three-dimensional and possibly singular interlaminar stress fields are to be expected, the plane-strain approach is upgraded by a layerwise displacement-based formulation wherein the physical laminate layers are discretized into a number of mathematical layers with respect to the thickness direction. The governing differential equations for the unknown additional displacement functions with respect to the width coordinate in the form of the Euler-Lagrange equations stemming from the underlying variational statement can be solved exactly and eventually lead to an eigenvalue problem that needs to be solved numerically. Usage of continuity conditions between the individual laminate layers and formulation of adequate boundary conditions at the free edges in an integral sense then lead to complete representations for displacements, strains and stresses at every location in the considered laminate. While the analysis approach relies on a discretization of the laminate into a number of mathematical layers with respect to the thickness direction and further requires a numerical solution of a quadratic eigenvalue problem, it provides closed-form analytical solutions concerning the width direction and can thus be classified as being a semi-analytical solution. The presented analysis method is compared to the results of comparative finite element simulations and is shown to be in good agreement, however with only a fraction of the computational effort that is required for according finite element simulations.

1. Introduction

Fiber-reinforced plastics (FRP) in the form of laminated plates or shells (i.e. flat or curved thin-walled structures that consist of a number of FRP layers, short: laminates) have found an increasing use in the last few decades in many diverse engineering applications due to their excellent properties in terms of a high strength-to-weight ratio and stiffness-to-weight ratio which makes such laminated composite structures especially attractive for applications where the weight of a structure is of utmost importance. Naturally, this holds for classical lightweight engineering structures as they are commonly treated in aeronautical engineering, but an increasing use of such materials can also be observed in other engineering branches such as the automotive industry or in the wind energy sector. A further advantage of laminated composite materials is the fact that due to the virtually infinite possibilities to design the stacking sequence of the individual laminate layers, it is possible to design laminates in such a way that the layups are optimally adjusted to the applied loadings and boundary conditions.

However, besides the obvious advantages of laminated plates and shells, there are certain disadvantages as well which need to be taken into account carefully during the design and analysis phase. One problem field that is entirely uncommon when dealing for instance with isotropic structures is the so-called free-edge effect which is a stress concentration phenomenon that mainly stems from the inherent anisotropy that characterizes fiber rein-

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forced materials and which needs to be taken into account carefully whenever laminated composite structures are being dealt with. The free-edge effect has been called to the attention of a broader scientific audience after the publication of the pioneering paper by Pipes and Pagano [1] in 1970 in which a plane laminated specimen under uniaxial tension has been treated by a finite difference formulation. Pipes and Pagano found that even though traction free edges of composite laminates may appear harmless on first sight when judged by engineering intuition, they nevertheless may be the location of delamination onset and subsequent propagation and thus failure of a laminate. This phenomenon, characterized by the occurrence of an inherently three-dimensional stress state dominated by interlaminar stresses (i.e. stresses in the thickness direction) which theoretically may even become singular when being dealt with in the framework of a macroscopic elasticity approach, is mainly due to the generally dissimilar elastic behaviour of adjacent laminate layers which is of course a disadvantage of such layered composite structures. The name of this phenomenon originated due to the fact that the three-dimensional stress state at the free edges is actually very localized and known to decay rapidly with increasing distance from the free edges.

Triggered by this first systematic work by Pipes and Pagano, the free-edge effect became the subject matter of an impressingly large number of investigations, covering topics such as the development of analysis methods (closed-form analytical, semi-analytical, numerical), the classification of the involved stress singularities, the optimization of laminated structures under consideration of free-edge effects, experimental procedures and data, the delamination onset and propagation at free laminate edges, up to related problems like similar stress concentrations at wedges, corners, holes, or notches in composite laminated structures. An encompassing literature survey is beyond the scope of this contribution, and a number of survey papers that summarize the state of the art in this field is available a selection of which is cited with [2–8]. The interested reader should consult these references for further information, also in order to appreciate the scientific progress in this field in a historical context.

Interlaminar stresses in laminated composite structures, however, are not only triggered at free laminate edges due to the incompatible elastic properties of adjacent dissimilar layers. Such stresses in the thickness direction may even be found in isotropic or non-layered anisotropic thin-walled shells wherein such thickness stresses can be the result of the curvature of the shell [9,10]. Even though these interlaminar stresses are usually not of a singular nature, they may become significant and thus need to be taken into account as well. Naturally, such stresses in the thickness direction may also appear in laminated structures remote from the free edges. The determination of stress fields in curved non-layered anisotropic and layered composite structures as well as in beams made of functionally graded materials and the experimental investigation of according specimens has been the subject matter in a number of publications, and a selective survey is given in the following. Blumer [11] investigated the stress fields in orthotropic curved beam structures made of glued laminated timber by means of adequate formulations for Airy's stress function and developed analysis equations that to this day are still widely used in civil engineering. An exact elasticity solution for the cylindrical bending of a curved isotropic beam with a Young's modulus that is a function of the radial coordinate has been presented by Lekhnitskii [12]. Tolf [13] developed a closed-form analytical solution for the bending of a cylindrically curved beam in a plane state of strain consisting of fiber-reinforced plastics. Within the analysis model, fibers and matrix were modeled as individual 'layers'. Tolf used adequate representations for the stress components as well as for the boundary and continuity conditions for the closed-form determination of all state variables in the considered beam structures. A similar analysis method applying Airy's stress function for each laminate layer subjected to plane stress conditions has been used by Ko and Jackson [14] who investigated cylindrically curved laminated beams subjected to end forces and moments. The solution presented in [14] will also be employed in this paper as a reference solution for the stress fields in the innermost laminate regions remote from the free edges. A similar investigation focussing on delamination stresses in curved composite laminated beams was presented by Ko [15]. Ko and Jackson extended their analysis approach to the analysis of horseshoe-shaped and elliptical composite structures under end forces in [16]. Kedward et al. [17] used the finite element method and an elasticity based approach for the determination of the interlaminar stresses and the assessment of the strength of curved composite structures. Kardomateas [18,19] developed analysis methods for the bending of cylindrically curved polar-orthotropic beams and generally anisotropic beams in which the elastic properties are linear functions of the radial coordinate. Vasilenko [20] presented an elasticity solution for layered curved beams in which the elastic properties of the individual layers were arbitrary functions of the radial coordinate. Shenoi and Wang [21] used an elasticity based approach for the determination of the stress fields in curved composite and sandwich beams that uses the results of a curved beam on an elastic foundation as a baseline. Wang and Shenoi [22] also employed an elasticity based approach for the determination of stress fields in curved sandwich beams using an analysis method based on Airy's stress functions for the stresses and studied the debonding and stability behaviour of the sandwich skin faces. Sharma and Bakis [23] presented a finite element study of the stress fields in C-shaped rings made of polar-orthotropic material. The distribution of the interlaminar normal stresses in curved composite laminates has been studied by Kress et al. [24] by extending an analysis model for thick-walled composite tubes and using a finite element procedure for the determination of free parameters in the analysis model. The thermal stresses in curved beams made of functionally graded materials with respect to the radial coordinate were investigated by Mohammadi and Dryden [25]. Experimental results for stresses and failure modes in curved composite structures were presented by e.g. Jackson and Martin [26], Shivakumar et al. [27], Cui et al. [28], Sharma and Bakis [23,29], or Avalon and Donaldson [30].

Even with the computer resources and computational power that are nowadays available, in an industrial environment there is still a considerable interest in the development of time-efficient closed-form approximate analysis methods without having to resort to numerical simulations every time an analysis task needs to be done. This especially holds true when engineers who are working on composite structures are confronted with tasks like structural optimization, conceptual assessments or extensive parametric studies during which one and the same analysis needs to be carried out hundreds or even thousands of times. To the best of the knowledge of the authors, up to this day there is no efficient approximate and yet reliable analysis method available that enables the assessment of free-edge stress fields in curved composite laminated structures. In order to contribute to this field, this paper includes an analysis approach for the determination of displacements, strains and stresses in symmetrically or unsymmetrically cylindrically curved cross-ply laminates that are clamped at one end and subjected to a bending moment at the other end. The analysis method that will be presented in the following consists of two parts, namely an 'inner' solution which is supposed to describe the stress state in the inner laminate regions remote from the free edges of the curved laminated composite structure, and a 'free-edge' solution that is designed to capture the free-edge effects that occur in the vicinity of the traction free edges. The 'inner' solution is based on the assumption of a plane strain state and uses an adequate layerwise representation for Airy's