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Three dimensional analysis of functionally graded plates up to yielding, using full layer-wise finite element method

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ABSTRACT

In this paper, full layer-wise method is employed to analyze the elastic bending of functionally graded plates up to yielding. The solution procedure is presented in two steps. Firstly, the elastic bending of simply supported FG plates is studied to find the deflection and stress distribution. Until this step, the mechanical properties are assumed to vary through the thickness following the rule of mixture. The numerical results are compared with those obtained from quasi 3D elasticity and various shear deformation theories and there has been an excellent agreement between them. In the next step, the mechanical properties of FG plates are determined by using TTO model as the modified rule of mixture. The volume fraction of material constituents is assumed to vary via a power-law function through the thickness in both steps. Finally, Tresca and Von Mises' yield criteria are utilized to predict the onset of yielding. The results obtained for four different boundary conditions and demonstrated that boundary conditions has a remarkable influence on the region in which yielding initiates. The effect of power-law index has been depicted on the onset of yielding as well.

1. Introduction

Engineering structures can be exposed to different types of loading such as bending, buckling and impact in accordance with their application fields. The ratio of strength to weight as one of the most important design parameters is desired to increase. Composite materials were introduced to achieve this purpose. However, these kinds of materials suffer from various modes of damage under loading such as delamination and deficiencies namely stress concentration at the interface of the layers. Functionally graded materials (FGMs) are a new class of composites which have been developed to resolve the deficiencies of conventional composite structures. Unlike conventional composite laminates in which the mechanical properties through the thickness change suddenly, the properties of functionally graded materials vary gradually and smoothly. Therefore, the residual stresses and stress concentration reduce significantly.

Recently, FGMs have been widely utilized in different structural applications such as aerospace, automotive, nuclear, etc. The integrity of functionally graded materials compared with conventional composite materials attracts many researchers' interests to develop new theories in order to analyze their mechanical behavior under different modes of loadings such as buckling, bending and vibration. Shear deformation theories [1–11] are the most common theories for investigating the elastic response of functionally graded plates. In all mentioned shear deformation studies, the governing equations were obtained by utilizing the principle of virtual work (PVW) in which only displacements are permitted to be considered as the primary variables. Brischetto and Carrera [12] demonstrated the outperformance of Reissner's mixed variational theorem (RMVT) over those governed by RVWs. In fact, in this method the shear and normal transverse stresses are also taken as the primary variables. A quasi-3D RMVT-based method is conducted by Wu et al. [13] to investigate the static behavior of simply supported FG plates. Another set of theories in which the effect of thickness stretching is considered during the procedure of solution is the quasi-3D theories. Neves et al. [14] developed a quasi-3D hyperbolic shear deformation theory to analyze the bending and free vibration of simply supported FG plates. In another similar approach, Neves et al. [15] implemented a quasi-3D sinusoidal shear deformation theory to carry out the static and free vibration analysis of FG plates. Thai and Kim [16] employed quasi-3D sinusoidal shear deformation theory to represent an analytical solution for predicting the bending behavior of simply supported FG plate. Asemi et al. [17] employed 3D elasticity theory to conduct static and dynamic analyses of fully clamped functionally graded skew plates. A three-dimensional low velocity impact analysis of functionally graded rectangular plates is performed by Asemi and Salami [18]. The influence of various parameters

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such as material inhomogeneity, projectile velocity and projectile density on the low velocity impact behavior of the FG plates has been investigated as well. Sedighi and Shakeri [19] presented a three-dimensional exact solution combined with Galerkin finite element method to study the electro-mechanical behavior of simply supported cylindrical shell panels composed of functionally graded piezoelectric materials (FGPM) subjected to pressure and electro-static excitation. In another similar three-dimensional approach Javanbakht et al. [20] studied the mechanical dynamic behavior of simply supported FGPM panels. Full layerwise finite element method as an efficient and flexible approach has been employed by Shakeri and Mirzaeifar [21] to investigate the static and dynamic analyses of functionally graded plates with surface bonded piezoelectric layers subjected to impulse loads. Two other layer-wise studies has been performed by Yas et al. [22,23] to investigate the dynamic response of functionally graded cylindrical shells and hollow thick cylinders made up of functionally graded materials with a piezoactuator ring.

Another approach for static and dynamic analyses of plates was introduced by Hughes et al. [24] named isogeometric analysis (IGA) which is based on Non-Uniform Rational B-Spline (NURBS) functions. This method has several advantages over conventional finite element analysis such as being capable of refining mesh via re-indexing parametric space with no interaction with Computer Aided Design (CAD) system. Yu et al. [25] employed this method for buckling analysis of functionally graded rectangular and skew plates with various boundary conditions subjected to thermo-mechanical loadings. In another similar approach, Yu et al. [26] developed this method to an extended isogeometric analysis (XIGA) for thermal buckling of functionally graded plates with internal defects such as cracks or cutouts. Yin et al. [27] employed FSDT-based IGA to investigate the effect of boundary conditions, geometrical shape, and gradient exponent on the bending, buckling, and free vibration of functionally graded and homogeneous plates. Liu et al. [28] performed a similar study by implementing quasi-3D elasticity based IGA considering thickness-stretching and shear deformation effects. A novel set of engineering results have been provided by Jin et al. [29] by representing 3D exact solution for free vibration analysis of functionally graded thick plates with classical and restrained boundary conditions. Utilizing Rayleigh-Ritz procedure to obtain the exact solution, they have also illustrated the effect of restrained boundary conditions on the vibrational behavior of plates. Other studies on free vibration of functionally graded structures investigating the influence of restrained boundary conditions on the final results have been conducted in the literature [30–33].

Although many efforts have been made to perform the elastic analysis of functionally graded materials, elastic-plastic analysis of these materials is remained a challenge to conduct. Numerous models have been developed to predict the mechanical properties of FGMs such as yield strength and Young's modulus. Tamura et al. [34] developed a modified rule of mixture named TTO[1] to predict the elastic behavior of cemented carbide composite materials. Bhattacharya et al. [35] found TTO model more accurate than the rule of mixture (ROM) for determining Young's modulus of two phased materials. They adopted experimental tests on four homogenized beams composed of Silicon-Carbide and Aluminum with different volume fractions. They generalized the results to functionally graded materials and utilized TTO model for approximating other mechanical properties as well. Huang and Han [36] employed Donnell shell theory and J_2 flow rule to analyze the elastic-plastic buckling of functionally graded cylindrical shells. In this approach, the Poisson's ratio was assumed to vary according to Voigt's rule of mixture and other mechanical properties were calculated by utilizing TTO model. Akis [37] provided an analytical solution to investigate the elastic-plastic behavior of FG spherical pressure vessels. Tresca's yield criterion was employed to distinguish the onset of yielding and the material is assumed to be ideally plastic. Other elastic-plastic analyses of functionally graded structures utilizing TTO model as the rule of mixture are available in the literature [38–43].

In the present study, full layer-wise finite element method is applied to study the elastic bending of functionally graded plates until yielding. By utilizing this method, the thickness stretching as well as shear deformation effects can be considered which makes this method more accurate than equivalent single layer theories. In other words, all six components of stresses which are necessary to calculate the equivalent yield stresses, can be obtained accurately by implementing layer-wise method. Additionally, layer-wise method has some advantages over conventional 3D-FEM. Firstly, the computational costs are remarkably lower due to less required degrees of freedom. The second merit that makes layer-wise method more efficient than 3D-FEM is the capability of changing the shape functions through the thickness independent of in-plane shape functions. The principle of minimum potential energy is employed to derive the field equations. A power-law function is assumed for the variation of material constituents through the thickness. The solution procedure is represented in two steps. Firstly, the elastic bending of simply supported FG plates subjected to transverse distributed load is investigated. Since there has been no study in the literature on bending analysis of rectangular plates made up of functionally graded materials utilizing TTO model, in the first step, in order to verify the results obtained from full layer-wise method, the mechanical properties of plates are assumed to vary according to ROM and the Poisson's ratio is considered to be constant. However, the simplifications assumed in the first step concluded unrealistic results. Moreover, Bhattacharya's researches illustrated that ROM is not an accurate model to determine the effective elastic properties of FGMs. In the next step, in order to eliminate these drawbacks, the mechanical properties such as Young's modulus and yield strength are assumed to vary in the direction of thickness following the TTO model. In addition, the Poisson's ratio is calculated by utilizing Voigt's rule of mixture. After deriving the governing equations and finding all stress components, the equivalent stresses are obtained with respect to Tresca and Von Mises' yield criteria. The onset of yielding, as one of the most essential design factors which has not been reported in the literature is predicted afterwards by comparing the equivalent stresses related to each node with the corresponding yield strength. The results are provided for FG plates composed of Silicone Carbide and Aluminum with various boundary conditions. The influence of boundary conditions and the power-law index on the region in which yielding initiates have also been depicted.

2. Mathematical formulation

Full layer-wise method which has been introduced by Reddy [44] for static and dynamic analysis of laminated composite plates is herein considered as the solution method. This method could be as accurate as 3D finite element method, while requires less operation during the procedure of governing field equations due to less degrees of freedom. This approach is an extension of layer-wise finite element method to FG plates. Therefore, the FG plate should be considered as a multilayer plate. Moreover, this method is a suitable choice for analyzing FG plates due to its flexibility of considering the variation of material properties through the thickness. In fact, layer-wise method is a combination of a 1D finite element through the thickness and a 2D finite element through the $x-y$ plane. The layer-wise element is obtained after assembling one element trough the thickness. Then the total stiffness matrix of the plate can be calculated by assembling the layer-wise element through the $(x-y)$ plane. This procedure is depicted in Fig. 1.

2.1. Kinematics

In order to apply the one-dimensional finite element, a shape

[1] TTO model was represented by Tamura, Tomato and Ozawa indicating the initials of their last names.