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GDQ analysis of a beam-plate with delaminations

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ABSTRACT

The buckling load and post-buckling deformation of a composite beam-plate having two non-overlapping delaminated regions under axial compression is studied by using the generalized differential quadrature (GDQ) method. Applying the GDQ method to each region and using the continuity conditions in transverse displacements, the slopes, the moments and shear forces for upper and lower parts at tips of delamination, the problem will transform to the generalized eigen problem. Various combinations of delaminated length, depth and locations to obtain the buckling load and post-buckling deformation are investigated.

1. Introduction

Delamination phenomenon of composite materials is the common failure mode in composite structures. Delamination occurs due to initial manufacturing imperfections or in service loads, which can significantly affect the strength and stiffness of laminated composite components. In 2016, Marjanović et al. [1] used the finite element (FE) method and the virtual crack closure technique (VCCT) to calculate the transient results of propagating delamination in laminated composite plates. Also the analytical solutions of double-cantilever-beam problem were obtained. In 2016, Li [2] used the extended layerwise method (XLWM) and VCCT to investigate the delamination in laminated composite plates and shells. The moving and growth of delaminations were also studied. There were some delamination studies about laminated plate under buckling and post-buckling loads. In 2016, Sreehari and Maiti [3] used the inverse hyperbolic shear deformation theory (IHSDT) and FEM to compute the delamination of damaged-laminated composite plates under thermo-mechanical loading. The delaminated behaviors of buckling and post buckling were studied. In 2016, Gan et al. [4] applied the FE model to investigate the ply-drop delamination onset in thick tapered composite beams. Parametric studies for the through-thickness location, drop thickness and the overall order of ply terminations are also investigated. In 2016, Li et al. [5] provided the analytical and experimental investigations for the active composite beams with delamination. Parametric studies e.g. the slope of deformed beam at the junction of delamination region, the length of the delamination are used to investigate the delaminated behavior of active beams. In 2016, Aslan and Daricik [6] use the experimental method to validate the multiple delaminations effects on the first critical buckling and re-buckling loads for E-glass/epoxy composites. The dominated influences of delaminations are the compressive, flexural strength and critical buckling load. In 2016, Kharghani and Soares [7] use an analytical method with a layerwise higher order shear deformation theory (HSDT) to study the behavior of laminated plate with embedded delaminations. The buckling load results of geometric and thickness of the delaminated area, debonding fracture are studied and compared with the FE analysis. In 2016, Hammami et al. [8] presented the experimental method on the vibration of glass fibre reinforced plastic (GFRP) composites with delaminations. Nonlinear elastic behavior for progressive delamination length is investigated. In 2015, Tornabene et al. [9] presented a review for the strong formulation finite element method (SFEM) which is a numerical method by using differential quadrature method (DQM) and the mapping technique.

Many studies of the behavior in rectangular plate and cylindrical shell have been presented by using the method of differential quadrature (DQ). In 2017, Tornabene et al. [10] presented the numerical techniques of SFEM and weak formulation finite element method (WFEM) and obtained the accurate and fast computing solutions. In 2014, Hong [11] used the generalized differential quadrature (GDQ) method to study the thermal vibration and transient response of magnetostrictive functionally graded material (FGM) plates. The computational GDQ results of parametric effect and mechanical boundary condition on the FGM provide some valuable data for the comparisons. In 2010, Hong [12] presented the GDQ computational method for the piezoelectric laminated cylindrical shell. The displacement and stress results at central position of composite shells with graphite–epoxy laminate and PVDF (polyvinyledene fluoride) surface layer are investigated under mechanical load and electric potential. In 2009, Hong [13] studied the transient responses of magnetostrictive plates with the GDQ method. The computational results of thermal stresses and center displacement with and without shear effect were investigated for the thin and thick laminated plate. In 1999, Moradi and Taheri used DQ

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method to solve the buckling load of a delaminated composite beam-plate [14]. In 1998, Hua and Lam used the GDQ method to study the effects of boundary conditions on the frequency characteristics for a thin rotating cylindrical shell [15]. In 1998, Liew and Teo used the DQ method to present the formulation and numerical analysis of the three-dimensional plate model [16]. In 1997, Shu and Du proposed the GDQ method to implement the clamped and simply supported boundary conditions for the free vibration analysis of beams and plates [17]. In 1989, Bert et al. investigated the behavior of thin rectangular orthotropic elastic plates with immovable edges and undergoing large deflections [18]. Beam-plates having two through-the-width delaminations resulting in one dimensional problem are considered in this study. In this research, the present study is interesting to obtain the solution of the post-buckling deformation of a delaminated beam-plate with two delaminations by using the GDQ method. The novelty of this paper is to provide an another good numerical GDQ buckling solution in the analysis of delaminated beam-plates.

2. Formulations

2.1. Governing equation

Consider the composite beam-plate with two clamped ends and two delaminated regions under compression. The delaminated regions are not overlapped as shown in Fig. 1. The delaminated beam-plate is divided into seven segments, and each segment has the same form of governing equation, which can be written as follow:

$$\frac{d^4w_k}{dx^4}+\lambda_k^2\frac{d^2w_k}{dx^2}=0,\quad k=1,2,...,7 \tag{1}$$

where w_k is the transverse deflection, P_k and D_k are the axial loading and the flexural rigidity of each section, respectively, $\lambda_k^2=P_k/D_k$, and x is the local coordinate of each section.

The clamped bondary conditions at both edges are assumed that $w_1(0)=0$, $w_1'(0)=0$, $w_7(l_7)=0$, and $w_7'(l_7)=0$, and to satisfy the continuity conditions in transverse displacement and slope for upper and lower parts at delaminated tips, one arrive at

$$w_1(l_1)=w_2(0)=w_3(0) \tag{2a}$$

$$w_1'(l_1)=w_2'(0)=w_3'(0) \tag{2b}$$

$$w_4(0)=w_2(a_1)=w_3(a_1) \tag{2c}$$

$$w_4'(0)=w_2'(a_1)=w_3'(a_1) \tag{2d}$$

and

$$w_4(l_4)=w_5(0)=w_6(0) \tag{3a}$$

$$w_4'(l_4)=w_5'(0)=w_6'(0) \tag{3b}$$

$$w_7(0)=w_5(a_2)=w_6(a_2) \tag{3c}$$

$$w_7'(0)=w_5'(a_2)=w_6'(a_2) \tag{3d}$$

The balance of the moments M and shear forces V at both junctions leads to the following equations:

The balance of shear forces at left junction is

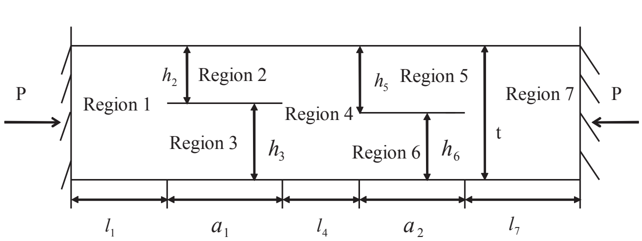

Fig. 1. Geometry of delaminated beam-plate.

$$V_1(l_1)=V_2(0)+V_3(0) \tag{4a}$$

$$V_4(l_4)=V_5(0)+V_6(0) \tag{4b}$$

The balance of shear forces at right junction is

$$V_4(0)=V_2(a_1)+V_3(a_1) \tag{5a}$$

$$V_7(0)=V_5(a_2)+V_6(a_2) \tag{5b}$$

The balance of moments at left junction is

$$M_1(l_1)-M_2(0)-M_3(0)+\frac{1}{2}P_3h_2-\frac{1}{2}P_2h_3=0 \tag{6a}$$

$$M_4(l_4)-M_5(0)-M_6(0)+\frac{1}{2}P_6h_5-\frac{1}{2}P_5h_6=0 \tag{6b}$$

The balance of moments at right junction is

$$M_4(0)-M_2(a_1)-M_3(a_1)+\frac{1}{2}P_3h_2-\frac{1}{2}P_2h_3=0 \tag{7a}$$

$$M_7(0)-M_5(a_2)-M_6(a_2)+\frac{1}{2}P_6h_5-\frac{1}{2}P_5h_6=0 \tag{7b}$$

in which the axial force P_k in each sub-laminate has its pre-buckled state, that is:

$$P_k=\frac{h_k}{t}P \tag{8}$$

where t is the total thickness of beam-plate.

2.2. GDQ method

Basically, the GDQ method can be restated that the derivative of a smooth function at a discrete point in a domain can be discretized by using an approximated weighting linear sum of the function values at all the discrete points in the direction [17]. The mth-order derivative of one-dimensional function $f(x)$ at the ith discrete point $x=x_i$ in the x direction is given by:

$$\frac{d\ ^mf(x)}{d\ x^m}|x=x_i=\sum_{j=1}^{N}C_{i,j}^{(m)}f(x_j),\quad i=1,2,...,N \tag{9}$$

where $C_{i,j}^{(m)}$ is the weighting coefficient related to the m^{th} -order derivative and N is the number of the total discrete grid points used in the x direction. They are given by the following formulas:

$$C_{i,j}^{(1)}=\frac{M^{(1)}(x_i)}{(x_i-x_j)M^{(1)}(x_j)}\quad \text{for } i\neq j \text{ and } i,j=1,2,...,N$$

$$C_{i,j}^{(m)}=m\left(C_{i,j}^{(1)}C_{i,i}^{(m-1)}-\frac{C_{i,j}^{(m-1)}}{x_i-x_j}\right)\quad for\ m=2,3,...,N-1, i\neq j \text{ and } i,j=1,2,...,N$$

$$C_{i,i}^{(m)}=-\sum_{j=1(j\neq i)}^{N}C_{i,j}^{(m)}\quad for\ i=1,2,...,N,\quad m=1,2,3,...,N-1 \tag{10}$$

$M^{(1)}(x)$ is the first derivative of $M(x)$ and they are defined as follows:

$$M(x)=\prod_{j=1}^{N}(x-x_j)\quad=(x-x_1)(x-x_2)...(x-x_N) \tag{11a}$$

$$M^{(1)}(x_k)=\prod_{j=1(j\neq k)}^{N}(x_k-x_j)\quad=(x_k-x_1)(x_k-x_2)...(x_k-x_N)|x_k\neq x_j \tag{11b}$$

and x_i is a discrete grid point, it can be arbitrarily chosen.

2.3. Discrete equations

Let h_k and l_k represent the thickness and length of each region, in which k= 1, 2,…, 7. Denoting that non-dimensional parameters: $X=x/l_k$, $W=w_k/h_k$ and applying the GDQ method to Eq. (1) for each