Composite Structures 182 (2017) 598–605

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Benchmark exact free vibration solutions for multilayered piezoelectric composite plates

F. Moleiro[a,*], A.L. Araújo[a], J.N. Reddy[b]

[a] LAETA, IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal
[b] Texas A & M University, College Station, TX 77483-3123, USA

ARTICLE INFO

Keywords:
Smart structures
Multilayered plates
Piezoelectric layers
Composite layers
Exact free vibration solutions

ABSTRACT

The development of advanced finite element models suitable for analysis of smart structures depends on a comprehensive understanding of the high layerwise inhomogeneity in mechanical and electric properties present in multilayered piezoelectric composite structures. The assessment of advanced models still continues to rely only on a few well-known benchmark three-dimensional (3D) exact solutions provided by pioneer works, namely, in the 1970s by Pagano as well as Srinivas for composite plates and later in the 1990s by Heyliger and Saravanos for piezoelectric composite plates. To overcome the limited number of test cases whose 3D exact solutions have been published, the method introduced by Heyliger to derive the 3D exact solutions has been successfully implemented using symbolic computing. New benchmark 3D exact free vibration solutions are provided for piezoelectric composite plates that complete a previous work with the static solutions. Two multilayered plates using PVDF piezoelectric material are chosen as test cases under three sets of electrical boundary conditions and considering three plate aspect ratios (from thick to thin plate), in a total of 18 test cases. For each case, the frequencies of the first six thickness modes are presented, along with some representative through-thickness mode shapes.

1. Introduction

Structures design technology is currently undergoing a groundbreaking change leading to a new generation of smart structures. The main goal of these multifunctional structures, wherein multiple properties of materials are taken advantage of, is that besides their major designated functionality, the same structural component may accomplish at least one more task, such as active vibration control, damping, noise attenuation, structural health monitoring, aeroelastic stability or shape change. A smart structure incorporates smart material sensors and actuators that enable it to monitor a change in its environment, whether external (such as loads or shape change) or internal (such as damage or failure), and adapt to it by modifying the system characteristics (such as stiffness or damping) or the system response (such as strain or shape), in a controlled manner. Among smart materials, piezoelectric sensors and actuators are being used extensively due to their excellent electromechanical properties and design flexibility, such that they can be easily integrated in multilayered composite structures. In fact, smart composite structures offer the possibility to combine the lightweight, superior mechanical and thermal properties of composite materials with actuation, sensing and control. Due to their self-monitoring and self-adaptive capabilities, smart composite structures technology has an enormous potential to improve high performance engineering applications.

Due to the presence of high layerwise inhomogeneity in mechanical and electric properties in these smart composite structures, an accurate electromechanical modelling requires a correct description of both mechanical and electrical fields, particularly in the thickness direction. This is precisely demonstrated by pioneer works on 3D exact solutions that became paramount to the understanding of multilayered piezoelectric composite structures. In the 1970s, Pagano [1,2] derived the 3D exact elasticity solutions for the static analysis of simply supported cross-ply composite laminates and sandwich plates (i.e. multilayered orthotropic plates). Likewise, in that time, Srinivas and Rao [3] developed the corresponding 3D exact elasticity solutions for the free vibration and bucking analyses. Later in the 1990s, Heyliger [4,5] extended the work of Pagano to include piezoelectric layers and provided the 3D exact solutions for the static analysis of simply supported multilayered orthotropic piezoelectric plates. Heyliger and Saravanos [6] also developed the corresponding 3D exact solutions for the free vibration analysis. All these 3D exact solutions have been extremely useful in assessing the accuracy of numerous laminated plate theories

* Corresponding author.
E-mail address: filipa.moleiro@tecnico.ulisboa.pt (F. Moleiro).

http://dx.doi.org/10.1016/j.compstruct.2017.09.035
Received 10 August 2017; Received in revised form 13 September 2017; Accepted 14 September 2017
Available online 18 September 2017
0263-8223/

and related finite element models. However, the limited number of test cases whose 3D exact solutions have been published has somewhat restricted the assessment of recent advanced models to the same few test cases. To overcome this shortcoming and in the interest of this emergent research area of modelling smart structures, the method introduced by Heyliger to derive the 3D exact solutions has been successfully implemented using symbolic computing. First, new additional benchmark 3D exact solutions for the static analysis of multilayered piezoelectric composite plates were provided in a previous work [7]. Now, the implemented exact solution method has been successfully extended to derive free vibration solutions as well. Hence, the present work aims to complete this previous one by providing new additional benchmark 3D exact solutions for the corresponding free vibration analysis of multilayered piezoelectric composite plates.

Since the turn of the century, a number of reviews and assessments on the modelling and analysis of multilayered structures have been available over time. Namely, the well-known review papers by Saravanos and Heyliger [8], Benjeddou [9] and Carrera [10,11], as well as the book of Reddy [12] devoted to composite structures, and more recently, the excellent review papers by Kapuria et al. [13] and Gupta et al. [14]. In overview, the models mostly differ in equivalent single layer or layerwise variable descriptions as well as in the chosen unknown variables, consistent with classical (i.e. displacement-based) or mixed formulations. Some of the most relevant contributions, within layerwise models most suitable for smart composite structures, are: the layerwise models developed by Heyliger, Saravanos and co-workers [15,16], the layerwise models of Semedo Garção et al. [17], the layerwise mixed models of Garcia Lage et al. [18], the classical and mixed models developed by Carrera and co-workers [19,20] using Carrera's unified formulation, and also the layerwise mixed least-squares models developed by Moleiro et al. [21–23]; all of which use the 3D exact solutions for assessment of the models predictive capabilities. More recently, advanced models especially devoted to free vibration analysis continue to be developed, such as the layerwise model of Araújo et al. [24] for smart sandwich plates with soft cores and the 3D model approach developed by Messina and Carrera [25,26]. Considering the stringent performance requirements of the new generation of smart composite structures, the free vibration analysis of multilayered structures is a key area of research. Hence, the importance of the benchmark 3D exact free vibration solutions here presented, which may prove to be quite useful and of particular interest for growing smart structures technology.

2. Governing equations for each layer

Consider a multilayered plate of total thickness h and rectangular planar geometry $a \times b$ made of N generally orthotropic layers, either piezoelectric or composite layers. A Cartesian coordinate system (x,y,z) is used for the multilayered plate, and similarly for each k-th layer, with the z-axis taken positive upward from the respective midplane along the thickness direction, as illustrated in Fig. 1.

In agreement with linear elasticity and piezoelectricity the complete set of linear governing equations for each general layer of the multilayered plate involves the following equations:

- Equations of motion and charge equation of electrostatics

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad \frac{\partial D_i}{\partial x_i} = 0 \tag{1}$$

Here σ_{ij} are the stress components, D_i are the electric displacement components and u_i are the displacement components, with $i,j = 1,2,3$. Thus, $u_1 = u$, $u_2 = v$ and $u_3 = w$, and likewise, x_i are the coordinates

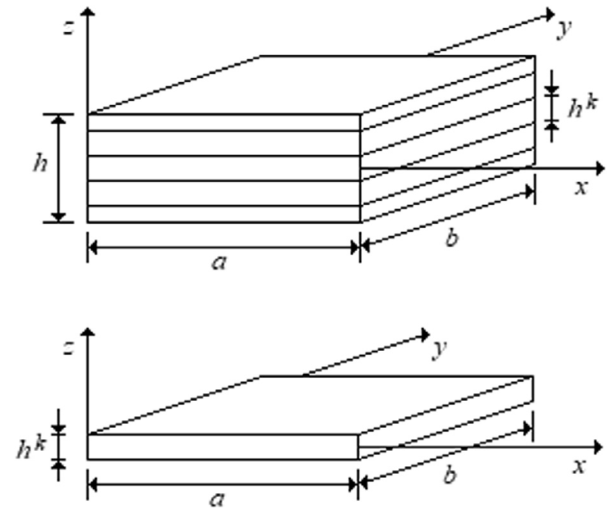

Fig. 1. Coordinate system for the multilayered plate and for each k-th layer.

with $x_1 = x$, $x_2 = y$ and $x_3 = z$.

- Constitutive equations

$$\sigma_i = C_{ij}\varepsilon_j - e_{ik}E_k, \quad D_k = e_{ki}\varepsilon_i + \epsilon_{kl} E_l \tag{2}$$

Here the standard engineering notation is used, and therefore, σ_i are the stress components and ε_i are the strain components, C_{ij} are the elastic stiffness coefficients, e_{ik} are the piezoelectric coefficients, E_k are the electric field components, D_k are the electric displacement components (as before) and ϵ_{kl} are the dielectric constants, with $i,j = 1,2,\ldots,6$ and $k,l = 1,2,3$.

- Strain-displacement equations and field-potential equations

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad E_k = -\frac{\partial \phi}{\partial x_k} \tag{3}$$

Here ε_{ij} are the strain components, ϕ is the electrostatic potential, E_k are the electric field components and u_i are the displacement components, with $i,j,k = 1,2,3$. Thus, as before, $u_1 = u$, $u_2 = v$ and $u_3 = w$, and x_i are the coordinates with $x_1 = x$, $x_2 = y$ and $x_3 = z$.

The expanded form of this complete set of governing equations can be found in Moleiro et al. previous works [22,23]. Note that in the layer constitutive equations C_{ij}, e_{ik} and ϵ_{kl} are already referred to the multilayered plate coordinate system (x,y,z), assuming that the transformation between the layer material coordinate system and the multilayered plate coordinate system is simply an in-plane rotation. See Reddy [12] for further details. Also, the 3D exact solutions as developed by Heyliger [4,5] assume that the most general conditions for the layer material properties are such that: the nonzero rotated elastic stiffness coefficients are C_{11}, C_{12}, C_{13}, C_{22}, C_{23}, C_{33}, C_{44}, C_{55} and C_{66}, the nonzero rotated piezoelectric coefficients are e_{31}, e_{32}, e_{33}, e_{24} and e_{15}, and the nonzero rotated dielectric constants are ϵ_{11}, ϵ_{22} and ϵ_{33}. Therefore, in respect to the material properties, the scope of the 3D exact solutions is limited to multilayered orthotropic piezoelectric plates.

In the end, substituting the constitutive equations, the strain-displacement equations and the field-potential equations into the three equations of motion and the charge equation gives four governing equations for each layer, in terms of the displacement components u, v and w and the electrostatic potential ϕ, as follows: